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TWO SERVER INTERDEPENDENT QUEUING MODEL USING B B S RULE

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Abstract— This paper explores a dual server queuing model with BBS rule, where the procedures for arrival and servicing are id. The model consists of two independent service facilities. The main objectives of this study are to determine the average queue length of the system and to obtain the average and standard deviation of the busy period distribution.
Keywords- Mutual influence, Waiting line system, Service mechanism, Arrival mechanism, bulk service, Mutual influence, Combined probability, marginal probabilities.

I. INTRODUCTION

The paper investigates a two-server id queueing model for analysis using BBS rule. This model extends the traditional two-server case. The BS process includes two service facilities with capacities B1 and B2, respectively.

A batch of B1 units or the whole queue length, whichever is shorter, is chosen for service from the head of the line whenever the first channel becomes available. Similarly, when the second channel becomes available, it takes B2 units (where B2 < B1) or the entire queue length (whichever is less) for service. When there is no queue length and both servers are idle, the next unit to arrive is always sent to the first service facility.

The arrival and service procedures are assumed to be id, and the two service facilities to function independently of one another. Such situations are common in places like Marshalling Yards with two engines or buildings with elevator systems having two lifts.

In the paper, the authors develop DDE and utilize GF methods to solve them. They derive and analyze the system characteristics in the presence of the DP. These models can be seen as generalizations of the BS queuing models presented by Arora in 1963.

II. TWO SERVER ID QUEUEING MODEL WITH B B S

In this study, we examine the B B S queueing model with two servers, where the arrival and service processes are id. The two service channels operate independently of each other. Poisson distribution governs the arrival process, which has a mean arrival rate of λ, additionally, the service time distributions in the two facilities are exponential, with mean service rates and respectively. Then the conditional process of the number of service completions of the first service facility given that the number of arrivals is of the form

$$P\lfloor X_{11} = n_1 / X_2 = n_2 : t \rfloor = e^{-(\mu_1 - \epsilon_1)t} \sum_{j=0}^{\min(n_1, n_2)} \left({}^{n_2}C_j \right) \left(\frac{\epsilon_1}{\lambda} \right)^j \left(\frac{\lambda - \epsilon_1}{\lambda} \right)^{n_2 - j} \left(\frac{(\mu_1 - \epsilon_1)t}{(n_1 - j)!} \right)^{n_1 - j} \quad ..(1)$$

here is the number of service completions in the first service facility during time t.
is the number of arrivals during time t, is the mean dependence rate.

Similarly, the quantity of services that the second service facility has completed given the number of arrivals is of the form,

$$P\{X_{12} = n_1 / X_2 = n_2 : t\} = e^{-(\mu_2 - \epsilon_2)t} \sum_{j=0}^{\min(n_1, n_2)} \left({}^{n_2}C_j \right) \left(\frac{\epsilon_2}{\lambda} \right)^j \left(\frac{\lambda - \epsilon_2}{\lambda} \right)^{n_2 - j} \left(\frac{(\mu_2 - \epsilon_2)t}{(n_1 - j)!} \right)^{n_1 - j} \quad ..(2)$$

Where, is the number of service completions in the second service facility during time t.
is the number of arrivals during time t, is the mean dependence rate.

Let the chance that at time t, the two channels are empty and no unit is in the queue and is waiting in the queue be . The chance that at time t, the first channel is busy and the second channel is empty be and no unit is in the queue. The chance that at time t, the first channel is empty and the second channel is busy be and no unit is waiting in the queue.

be the probability that at time t and , either of the two channels is busy and there is no unit waiting in the queue. And at time t, be the probability, both the channels are busy and there are n (be the number of units waiting in the queue.

The D.D.E of the model, with the above probabilities, are

$$P_{00}'(t) = \{-(\lambda - \mu)P_{00}(t)\} + \{(\mu_1 - \epsilon_1)P_{10}(t)\} + \{(\mu_2 - \epsilon_2)P_{11}(t)\}$$

$$P_{10}'(t) = \{-(\lambda + \mu_1 - \epsilon - \epsilon_1)P_{10}(t)\} + \{(\lambda - \epsilon)P_{00}(t)\} + \{(\mu_2 - \epsilon_2)P_0(t)\}$$

$$P_{01}^{/}(t) = -\{(\lambda + \mu_2 - \in - \in_2) P_{01}(t)\} + \{(\mu_1 - \in_1) P_0(t)\}$$

$$P_0^{/}(t) = \{-(\lambda + \mu_1 + \mu_2 - 2\in) P_0(t)\} + \left\{(\mu_1 - \in_1)\sum_{i=1}^{b_1} P_i(t) + (\mu_2 - \in_2)\sum_{j=1}^{b_2} P_j(t)\right\} + \{(\lambda - \in)[P_{10}(t) + P_{01}(t)]\}$$

$$P_n^{/}(t) = -(\lambda + \mu_1 + \mu_2 - 2\in) P_n(t) + (\lambda - \in) P_{n-1}(t) + (\mu_1 - \in_1) P_{n+B_1}(t) + (\mu_2 - \in_2) P_{n+B_2}(t)$$

For $n \geq 1$ Where $\in = \in_1 + \in_2$ ......(3)

$P_{00}(0) = 1$, $P_{10}(0) = 0$, $P_{01}(0) = 1$ and $P_n(0) = 0$ With the initial conditions,

Let the generating function of $P_n(t)$ be $F(x,t)$

$$F(x,t) = \sum_{n=0}^{\infty} P_n(t)\, x^n \qquad ......(4)$$

Multiplying equation (3) by proper powers of *x* and add, we get

$$\frac{\partial F(x,t)}{\partial t} = \left\{-(\lambda + \mu_1 + \mu_2 - 2\in) - (\lambda - \in)x - \frac{(\mu_1 - \in_1)}{x^{B_1}} - \frac{(\mu_2 - \in_2)}{x^{B_2}}\right\} F(x,t)$$

$$+ (\mu_1 - \in_1)\sum_{i=0}^{B_1 - 1}(1 - x^{i - B_1}) P_i(t) + (\mu_2 - \in_2)\sum_{j=0}^{B_2 - 1}(1 - x^{i - B_2}) P_j(t)$$

$$-(\mu_1 + \mu_2 - \in) P_0(t) + (\lambda - \in)[P_{10}(t) + P_{01}(t)] \quad ......(5)$$

Applying Laplace-transformation to the equations (3) and (5) and using the initial conditions, we have

$$-(s + \lambda - \in) P_{00}^{*}(t) + (\mu_1 - \in_1) P_{10}^{*}(t) + (\mu_2 - \in_2) P_{01}^{*}(t) = 0$$

$$-(s + \lambda + \mu_1 - \in - \in_1) P_{10}^{*}(t) + (\lambda - \in) P_{00}^{*}(t) + (\mu_2 - \in_2) P_0^{*}(t) = 0$$

$$-\left(s+\lambda+\mu_2-\in-\in_2\right)P_{01}^*(t) + \left(\mu_1-\in_1\right)P_0^*(t)=0 \quad \ldots\ldots(6)$$

$F^*(x,s)$ =

$$x^{B_1}\left\{\left(\mu_1-\in_1\right)\sum_{i=0}^{B_1-1}\left(1-x^{i-B_1}\right)P_i^*(s)+\left(\mu_2-\in_2\right)\sum_{j=0}^{B_2-1}\left(1-x^{j-B_2}\right)P_j^*(s)-\left(\mu_1+\mu_2-\in\right)P_0^*(s)+\left(\lambda-\in\right)\left[P_{10}^*(s)+P_{01}^*(s)\right]\right\}\div$$

$$\left\{-(\lambda-\in)x^{B_1+1}+\left(s+\lambda+\mu_1+\mu_2-2\in\right)x^{B_1}-\left(\mu_2-\in_2\right)x^{B_1-B_2}-\left(\mu_1-\in_1\right)\right\} \quad \ldots\ldots(7)$$

Applying Roche's theorem, it can be seen that for B_2

$$\left\{-(\lambda-\in)x^{B_1+1}+\left(s+\lambda+\mu_1+\mu_2-2\in\right)x^{B_1}-\left(\mu_2-\in_2\right)x^{B_1-B_2}-\left(\mu_1-\in_1\right)\right\}=0$$

. . . (8)

Has B_1 root's inside the unit circle $|x|=1$. Let the roots that lies inside $|x|=1$ be denoted by $x_k(s)$, $k=$1, 2 , 3 .. B_1 and that which lies outside be $x_0(s)$.

Since $F^*(x,s)$ is regular inside $|x|=1$ and the denominator has got B_1 zero's inside unit circle. The numerator must vanish at those zeros giving rise to B_1 linear equations. They are

$$\left(\mu_1-\in_1\right)\sum_{i=0}^{B_1-1}\left(1-x_k^{i-B_1}(s)\right)P_i^*(s)+\left(\mu_2-\in_2\right)\sum_{j=0}^{B_2-1}\left(1-x_k^{j-B_2}(s)\right)P_j^*(s)$$

$$-\left(\mu_1+\mu_2-\in\right)P_0^*(s)+\left(\lambda-\in\right)\left[P_{10}^*(s)+P_{01}^*(s)\right]\Big\}=0 \ \ldots\ldots\ldots\ldots(9) \quad \text{For } k=1,2,3,\ldots\ b_1$$

in (B_1+2) unknowns. Out of these, $P_{10}^*(s)$ and $P_{01}^*(s)$ could be expressed in terms of $P_0^*(s)$. From equation (6) and have these set of equations (9) would contain unknowns in $P_n^*(s)$, *n* = 0, 1, 2, . . . (B_1-1) and as such the unknowns would be determined completely. We can express $F^*(x,s)$, since the degree of the denominator is more than the numerator by one, in terms of the exterior zero as

$$F^{*}(x,s)=\frac{A(s)}{x_{0}(s)-x} \quad ....(10)$$

Therefore, $P_{n}^{*}(s)=\frac{A(s)}{x_{0}^{n+1}(s)}$; for $n \geq 0$

$$P_{0}^{*}(s)=\frac{A(s)}{x_{0}(s)} \quad ....(11)$$

Here $A(s)$ is an unknown to be determined. Solving equation (6) using equation (11) and writing $x_{0}(s)$ as x_{0}, we get

$$P_{00}^{*}(s)=\Big\{(s+\lambda-\in-\in_{1})+(\mu_{1}-\in_{1})(\mu_{2}-\in_{2})A(s)\times$$
$$\big[2(s+\lambda-\in)+(\mu_{1}+\mu_{2}-\in)\big]\big[x_{0}(s+\lambda+\mu_{2}-\in-\in_{2})\big]^{-1}\Big\}\div$$

$$\{(s+\lambda-\in)(s+\lambda+\mu_{1}-\in-\in_{1})-(\lambda-\in)(\mu_{1}-\in_{1})\} \quad ....(12)$$

$$P_{10}^{*}(s)=\Big\{(\lambda-\in)+(\mu_{2}-\in_{2})A(s)\times$$
$$\big[(s+\lambda-\in)(s+\lambda+\mu_{2}-\in-\in_{2})+(\lambda-\in_{1})(\mu_{1}-\in_{1})\big]\times$$
$$\big[x_{0}(s+\lambda+\mu_{2}-\in-\in_{2})\big]^{-1}\Big\}\div$$
$$\{(s+\lambda-\in)(s+\lambda+\mu_{1}-\in-\in_{1})-(\lambda-\in)(\mu_{1}-\in_{1})\} \quad ....(13)$$

$$P_{10}^{*}(s)=(\mu_{1}-\in_{1})A(s)\,\{x_{0}(s+\lambda+\mu_{2}-\in-\in_{2})\}^{-1} \quad ....(14)$$

From the equations (7) and (10) by putting $x=1$, we get,

$$\frac{A(s)}{x_{0}-1}=F^{*}(x,s)=\Big\{-(\mu_{1}+\mu_{2}-\in)P_{0}^{*}(s)+(\lambda-\in)\big[P_{10}^{*}(s)+P_{01}^{*}(s)\big]\Big\}\div s \quad ....(15)$$

Substituting equations (11), (13) and (14) in (15), we get,

$$A(s)=\left\{(\lambda-\in)^2 x_0\left(x_0-1\right)\left(s+\lambda+\mu_2-\in-\in_2\right)\right\}\div$$

$$\left[s\, x_0\left(s+\lambda+\mu_2-\in-\in_2\right)\right]$$
$$\left[(s+\lambda-\in)\left(s+\lambda+\mu_1-\in-\in_1\right)-(\lambda-\in)\left(\mu_1-\in_1\right)\right]\ -\left(x_0-1\right)$$
$$\left[(\lambda-\in)(s+\lambda-\in)(\mu_1-\in_1)\left(s+\lambda+\mu_1-\in-\in_1\right)\left(\mu_2-\in_2\right)\left(s+\lambda+\mu_2-\in_2-\in\right)\right]$$

$$-\left\{(\lambda-\in)^2\left(\mu_1-\in_1\right)\left(\mu_1-\in_1-\mu_2-\in_2\right)\right\}-\left(\mu_1-\in_1-\mu_2-\in_2\right)$$

$$\left(s+\lambda+\mu_2-\in-\in_2\right)\left\{(s+\lambda-\in)\left(s+\lambda+\mu_1-\in-\in_1\right)-(\lambda-\in)\left(\mu_1-\in_1\right)\right\}\quad \ldots\ldots(16)$$

Substituting the values of A(s) from equation (16) into equations (11) to (14) gives the Laplace-transform of all the probabilities defined earlier. To simplify the mathematical complexity, we take $\mu_1=\mu_2=\mu$ and $\in_1=\in_1=\frac{\in}{2}$, then the equation (11) reduces to

$$P_n^*(s)=\left\{(\lambda-\in)^2\left(x_0-1\right)\right\}\div\left\{s\, x_0^n\left[(s+\lambda-\in)+\left(s+\lambda+\mu-3\frac{\in}{2}\right)x_0-(\lambda-\in)\left(\mu-\frac{\in}{2}\right)\right]\right.$$

$$\left.+(2\mu-\in)\left(x_0-1\right)\left(s+\lambda+\mu-3\frac{\in}{2}\right)\right\}\quad \ldots\ldots\ldots(17)$$

In the queue, the average number of customers is L_q in steady state, then by using the Tauberian theorem, namely,

$\lim\limits_{s\to 0}\left[s\, P_n^*(s)\right]=\lim\limits_{t\to\infty}\left[P_n(t)\right]$, if the right limit exist, then

$$L_q=\sum_{n=1}^{\infty} n\left\{s\, P_n^*(s)\right\}_{s=0}=2\left(\frac{(\lambda-\in)}{2\mu-\in}\right)^2 x_0'\div\left\{\left[2\left(\frac{\lambda-\in}{2\mu-\in}\right)^2 x_0'+\left(1+\frac{2(\lambda-\in)}{2\mu-\in}\right)\left(x_0'-1\right)\right]\left(x_0'-1\right)\right\}$$
........(18)

Where x_0' is the root of the equation given in equation (8),

When $B_1=B_2=1$ then, $x_0'=2\frac{\mu}{\lambda}$ and the average queue length is

$$L_q = \frac{2(\lambda-\in)^3(2\mu-\in)}{(2\mu-\in)^2-4(\lambda-\in)^2} \qquad ....(19)$$

Table 1. Values of L_q

$B_1 = 5, \quad \lambda = 2$ and $\mu = 3$

B_2	$\in$				
	0	0.2	0.4	0.6	0.8
0	0.1229	0.10132	0.08091	0.06181	0.0446
1	0.06056	0.0497	0.03941	0.03002	0.0216
2	0.05239	0.04337	0.03476	0.02669	0.01941
3	0.05081	0.044219	0.03391	0.0261139	0.0191
4	0.05048	0.04192	0.03341	0.02603	0.0190

From the above table for clipped values of B_1 , λ , γ and varying values of $\in$ the values of L_q are computed. It is observed that $\in$ increases then the mean queue length L_q decreases. And also the average queue length L_q decreases as B_2 increases for fixed values of λ , γ and B_1 and the DP.

Busy Period Analysis:

We determine the busy period distribution for two cases, namely,

i) One or more of the servers is still in use and

ii) The two servers are still in use.

A single channel is occupied.: When both the servers in the channels are idle, at least one of the server busy with the arrival of a unit, and this lasts up to the instant at which both channels are become idle (i.e., the busy period starts with the arrival of a unit).

We assume

$B_1 = B_2 = b;$

$\mu_1 = \mu_2 = \mu ;$

$\in_1 = \in_2 = \in/2$

$P_{11}(t) = \{P_{10}(t) + P_{01}(t)\}$

The required probability function is given by

$$\gamma_1(\mathrm{t}) = \frac{d}{dt}\{P_{00}(\mathrm{t})\} \qquad \ldots\ldots(20)$$

And the corresponding equations are

$$\frac{d}{dt}P_{00}(\mathrm{t}) = \left(\mu - \frac{\in}{2}\right)P_{11}(\mathrm{t})$$

$$\frac{d}{dt}P_{11}(\mathrm{t}) = -\left(\lambda + \mu - 3\frac{\in}{2}\right)P_{11}(\mathrm{t}) + (2\mu - \in)P_0(\mathrm{t}) \qquad \ldots\ldots(21)$$

$$F^*(x,s) = x^s\left\{(2\mu-\in)\sum_{i=0}^{b-1}(1-x^{1-s})P_1^*(s) - (2\mu-\in)P_0^*(s) + (\lambda-\in)P_{11}^*(s)\right\} \div$$
$$\left\{(\lambda-\in)x^{s+1} - (s+\lambda+2\mu-2\in)x^s + (2\mu-\in)\right\} \qquad \ldots\ldots(22)$$

As before, let

$$F^*(\mathrm{x,s}) = \mathrm{B(s)}/(\mathrm{x}_0 - x)$$

And $F^*(1,\mathrm{s}) = \dfrac{\mathrm{B(s)}}{(\mathrm{x}_0 - 1)} = \left[(\lambda-\in)P_{11}^*(s) - (2\mu-\in)P_0^*(s)\right] \div s$

Substituting the values of

$$P_{11}^*(s) = (s+\lambda+\mu-3\in/2)^{-1}\left[1 + +(2\mu-\in)P_0^*(s)\right]$$

$$P_0^*(s) = \frac{\mathrm{B(s)}}{x_0}$$

From this equation, we get B(s) as

$$B(s) = \left[(\lambda-\in)x_0(x_0-1)\right] \div \left[sx_0(s+\lambda+\mu-3\in/2) + (2\mu-\in)(x_0-1)(s+\mu-\in/2)\right] \quad \ldots.(23)$$

Then,

$$\gamma_1^* = P_{00}^*(s) = \left(\mu - \frac{\in}{2}\right)P_{11}^*(s)$$
$$= \left[(2\mu-\in)\,x_0(s+2\mu-\in) - (2\mu-\in)^2\right] \div$$
$$2\left[\{(s+\lambda+\mu-3\in/2)s + (2\mu-\in)(s+\mu-\in/2)\}\,x_0 - (2\mu-\in)(s+\mu-\in/2)\right]$$

$$\ldots.(24)$$

This can be expressed as

$$\gamma_1^*(s) =$$
$$= \frac{\left[(2\mu-\in)\{(s+2\mu-\in)\}x_0 - (2\mu-\in)^2\right]}{2\{s(s+\lambda+\mu-3\in/2) + (2\mu-\in)(s+\mu-\in/2)\}x_0}\sum_{i=0}^{\infty}\left[\frac{(2\mu-\in)(s+\mu-\in/2)}{\{s(s+\lambda+\mu-3\in/2) + (2\mu-\in)(s+\mu-\in/2)\}x_0}\right]^i$$

$$\ldots.(25)$$

where x_0 is the root of the equation,

$$(\lambda-\in)x^{s+1} - (s+\lambda+2\mu-2\in)x^2 + (2\mu-\in) = 0$$

such that $|x_0| > 1$ $\qquad \ldots.(26)$

Following the heuristic argument of Arora(1963) the moments of the distribution are obtained b differentiating equation (24) and setting *s = 0.*

The average and standard deviation are obtained as

Average = $\dfrac{\left\{\left[1+\dfrac{(\lambda-\in)}{2\mu-\in}\right]y_0-1\right\}}{[(\mu-\in/2)(y_0-1)]}$ And

Variance =

$$\left\{\frac{(\lambda-\in)}{(2\mu-\in)}(s+1)\left[1+\frac{3(\lambda-\in)}{(2\mu-\in)}+\left[\frac{(\lambda-\in)}{(2\mu-\in)}\right]^2\right]y_0^3-\left[1+\frac{(\lambda-\in)}{(2\mu-\in)}\right]\left[s\left[1+\frac{5(\lambda-\in)}{(2\mu-\in)}+\left[\frac{(\lambda-\in)}{(2\mu-\in)}\right]^2+\frac{2(\lambda-\in)}{(2\mu-\in)}\right]y_0^2\right]\right.$$

$$\left.+\left[\frac{(\lambda-\in)}{(2\mu-\in)}(s+2)+2s\left[1+\frac{(\lambda-\in)}{(2\mu-\in)}\right]^2\right]y_0-s\left[1+\frac{(\lambda-\in)}{(2\mu-\in)}\right]\right\}$$

$$\left\{\left(\mu-\frac{\in}{2}\right)^2(y_0-1)^2\left[\frac{(\lambda-\in)}{(2\mu-\in)}(s+1)\,y_0-\left[1+\frac{(\lambda-\in)}{(2\mu-\in)}\right]s\right]\right\}^{-1}$$

Where $y_0>1$, is the root of the equation (26) . . . . . .(27)

In particular case when s=1 in equation (27), we get

Average = $\dfrac{s}{(2\mu-\lambda)}$ and variance = $\dfrac{4(2\mu-\in)}{(2\mu-\lambda)^3}$ and

Standard deviation = $2\sqrt{\dfrac{(2\mu-\in)}{(2\mu-\lambda)^3}}$ . . . . . .(28)

Both channels are busy:

The probability density for both the channels are busy is

$\gamma_2(\mathrm{t})=\dfrac{d}{dt}V(\mathrm{t})$

The equation which determines the probability density are

$$d\{P_{11}(t)\}=\left(\mu-\frac{\in}{2}\right)P_0(t)$$

$$d\{P_0(t)\}=-(\lambda+2\mu-2\in)P_0(t)+(2\mu-\in)\sum_{i=1}^{s}P_i(t)$$

$$\frac{d}{dt}\{P_n(t)\}=-(\lambda+2\mu-2\in)P_n(t)+(\lambda-\in)P_{n-1}(t)+(2\mu-\in)P_{n+s}(t) \quad \text{for } n\geq 1 \quad ..(29)$$

Under the initial conditions, then $L\{F(x,t)\}$ is

$$F^*(x,s)=-\frac{\left\{x^s+(2\mu-\in)\sum_{i=0}^{s-1}(x^s-x^i)P_i^*(s)-(2\mu-\in)x^sP_0^*(s)\right\}}{\{(\lambda-\in)x^{s+1}-(s+\lambda+2\mu-2\in)x^s+(2\mu-\in)\}} \quad .....(30)$$

Following the same argument as given in the previous section. $F^*(x,s)$ can be expressed in this form

$$F^*(x,s)=\frac{c(s)}{(x_0-x)} \quad ......(31)$$

Letting x=1 in equation (3.11) the solving for $c(s)$, we get

$$c(s)=\frac{x_0(x_0-1)}{(s+2\mu-\in)x_0-(2\mu-\in)} \quad \ldots\ldots(32)$$

Thus

$$\bar{\gamma}_2(s)=\frac{(2\mu-\in)(x_0-1)}{(s+2\mu-\in)x_0-(2\mu-\in)} \quad \ldots\ldots(33)$$

$$\text{Average}=\frac{1}{(2\mu-\lambda)} \qquad \text{variance}=\frac{(\lambda+2\mu-2\in)(2\mu-\in)}{(2\mu-\lambda)^3} \qquad \text{and}$$

$$\text{Standard deviation}=\sqrt{\frac{(\lambda+2\mu-2\in)(2\mu-\in)}{(2\mu-\lambda)^3}} \quad \ldots\ldots(34)$$

When $B_1 = 1$, $B_2 = 0$, $\mu_1 = \mu_2$, $\in = 0$ then it becomes to that of classical M/M/1 model.

This model is equivalent to the model (21) when $B_2 = 0$

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BBS: Baileys Bulk Service
id = interdependent
BS=Bulk Service
DDE=Difference-Differential Equations
GF=generating function
DP=dependence parameter