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STRONGLY $\xi^*_{\mathfrak{I}}$-SEMI-CONTINUOUS MAPS ON $I\xi_T S$

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Abstract: *In this paper, we introduced the several generalized forms of continuous maps such as strongly $\xi^*_{\mathfrak{I}}$-semi-continuous maps and strongly$\xi^*_{\mathfrak{I}}$-pre-continuous maps in generalized binary ideal topological spaces and investigate various relationships of these maps by making the use of some counter examples.*

Keywords: Stronglyξ-continuous, strongly $\xi^*_{\mathfrak{I}}$-semi-continuous, strongly $\xi^*_{\mathfrak{I}}$-pre-continuous

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1. INTRODUCTION

Information systems are fundamental instruments for generating information understanding in any real-life sector, and topological information collection structures are appropriate mathematical models for both quantitative as well as the qualitative information mathematics. This has an impact in digital topology and computer science

Kuratowski (1930) introduced the concept of ideal topological space. The concepts of semi-open and semi-continuous maps is introduced and studied by Levine (1963). Jankovic and Hamlett (1992) studied the concept of a local function and obtained the significant properties of these functions. Meanwhile, Dontchev (1996) verified the certain properties of pre-I-open sets. Hatir and Noiri (2006) and verified several properties of β−I-open sets and studied the several results of almost-I-continuities. Jafaril and Rajesh (2011) studied the concept of g-closed sets with respect of ideals and studied various characterizations.

Levine (1961) introduced weakly continuous functions and established some new results. Further, Son, Park and Lim (2007) introduced weakly clopen and almost clopen functions and investigate various properties of almost clopen functions. Nithyanantha and Thangavelu (2011) studied binary topology and investigate various characterizations. Nazir Ahmad Ahengar, Arvind Kumar Sharma, et.al (2022) introduced and studied the concept of some **ξ**-pre-continuous maps and establish the various relationships. Further Nazir Ahmad Ahengar, Nishi Gupta et.al (2023) introduced and studied the concept of strongly $\xi_{\mathfrak{I}}$-continuous maps in Binary ideal topological spaces

In this paper we developed the concept of strongly ${\xi^*}_{\mathfrak{I}}$-semi-continuous and strongly ${\xi^*}_{\mathfrak{I}}$-pre-continuous maps. Some require basic definitions, concepts of ξ-topological, $I\xi_T S$ and notations are discussed in Section 2. The concept of strongly ${\xi^*}_{\mathfrak{I}}$-semi-continuous map is discussed in Section 3. The conclusion is given in Section 4.

2. PRELIMINARIES

In this portion, we discussed few require and important definitions, concepts of ξ-topological, $I\xi_T S$ and some notations.

Definition 2.1: SupposeΥ_1 and Υ_2 are any two non-void sets. Then ξ_T from Υ_1 to $\Upsilon_2$is a binary structure $\xi \subseteq \wp(\Upsilon_1) \times \wp(\Upsilon_2)$ satisfying the conditions i.e., $(\emptyset, \emptyset)$,$(\Upsilon_1, \Upsilon_2) \in \xi$ and If $\{(L_\alpha, M_\alpha) ; \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha , \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from $\Upsilon_1$to Υ_2, then $(\Upsilon_1, \Upsilon_2, \xi)$ is known as ξ-topological space ($\xi_T S$) and the elements of ξ are known as the ξ-open subsets of $(\Upsilon_1, \Upsilon_2, \xi)$. The elements of $\Upsilon_1 \times \Upsilon_2$ are known as ξ-points.

Definition 2.2: LetΥ_1 and Υ_2 be any two non-void set and (L_1, M_1), (L_2, M_2) are the elements of $\wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$and $M_1 \subseteq M_2$.

Definition 2.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be a $\xi_T S$ and $L \subseteq \Upsilon_1, M \subseteq \Upsilon_2$. Then (L, M) is called ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ if $(\Upsilon_1 \backslash L, \Upsilon_2 \backslash M) \in \xi$.

Definition 2.4: Any non-empty collection $\mathfrak{I}$ of subsets of $\Upsilon_1 \times \Upsilon_2$ is an ideal only if it satisfies the two important axioms, i.e. if $(L, M) \in \mathfrak{I}$ and $(P, Q) \subseteq (L, M)$ then$(P, Q) \in \mathfrak{I}$ and If $(L, M) \in \mathfrak{I}$ and $(P, Q) \in \mathfrak{I}$ then $(L \cup P,\ M \cup Q) \in \mathfrak{I}$. Let ξ be ξ_T and $\mathfrak{I}$ be an ideal, then $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ is said to be an ideal ξ-topological space ($I\xi_T S$).

Definition 2.5: Let $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq \Upsilon_1 \times \Upsilon_2$. Then the set $(L, M)^*(\mathfrak{I}) = \{(x, y) \in \Upsilon_1 \times \Upsilon_2 / (U \cap L, V \cap M) \notin \mathfrak{I}$ for every nbd (U, V) of $(x, y)\}$ is known as the local function of (L, M) in the respect of $\mathfrak{I}$ and ξ. We normally denote $(L, M)^*$ instead of $(L, M)^*(\mathfrak{I})$ to avoid any confusion.

Definition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq \Upsilon_1 \times \Upsilon_2$. Then (L, M) is known as $\xi_{\mathfrak{I}}$-semi-open if for anyξ-open set (U, V),$(L, M) \backslash Cl_\xi(U, V) \in \mathfrak{I}$ whenever,$(U, V) \backslash (L, M) \in \mathfrak{I}$. Likewise$(L, M)$ is known as$\xi_{\mathfrak{I}}$-α-openif for anyξ-open set (U, V),$(L, M) \backslash I_\xi(Cl_\xi(U, V)) \in \mathfrak{I}$ whenever, $(U, V) \backslash (L, M) \in \mathfrak{I}$.

Definition 2.7: Let $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq \Upsilon_1 \times \Upsilon_2$. Then (L, M) is known as $\xi_{\mathfrak{I}}$-pre-open if for any ξ-open set (U, V), $(U, V) \backslash Cl_\xi(L, M) \in \mathfrak{I}$whenever, $(L, M) \backslash (U, V) \in \mathfrak{I}$. Likewise

(L,M) is known as$\xi_{\mathfrak{J}}$-β-openif for any ξ-open set (U,V) such that $(\mathrm{U},\mathrm{V})\backslash \mathrm{I}_{\xi}(\mathrm{Cl}_{\xi}(\mathrm{L},\mathrm{M})) \in \mathfrak{J}$ whenever, $(\mathrm{L},\mathrm{M})\backslash(\mathrm{U},\mathrm{V}) \in \mathfrak{J}$.

Definition 2.8: Let $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ be $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$ and $(\mathrm{L},\mathrm{M}) \subseteq \Upsilon_1 \times \Upsilon_2$. Then (L,M) is called ${\xi^*}_{\mathfrak{J}}$-open set if $(\mathrm{U},\mathrm{V})\backslash(\mathrm{L},\mathrm{M}) \in \mathfrak{J}$ whenever,$(\mathrm{L},\mathrm{M}) \subseteq (\mathrm{U},\mathrm{V})$, where (U,V) is ξ-open set.

Definition 2.9: Let $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ be $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$ and $(\mathrm{L},\mathrm{M}) \subseteq \Upsilon_1 \times \Upsilon_2$. Then (L,M) is called ${\xi^*}_{\mathfrak{J}}$-semi-open set if $(\mathrm{U},\mathrm{V})\backslash \mathrm{I}_{\xi}(\mathrm{L},\mathrm{M}) \in \mathfrak{J}$ whenever,$(\mathrm{L},\mathrm{M}) \subseteq (\mathrm{U},\mathrm{V})$, where (U,V) is ξ-open set. Similarly(L,M) is called ${\xi^*}_{\mathfrak{J}}$-α-open set if $(\mathrm{U},\mathrm{V})\backslash \mathrm{Cl}_{\xi}(\mathrm{I}_{\xi}(\mathrm{L},\mathrm{M})) \in \mathfrak{J}$ whenever,$(\mathrm{L},\mathrm{M}) \subseteq (\mathrm{U},\mathrm{V})$, where (U,V) is ξ-open set.

Definition 2.10: Let $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ be $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$ and $(\mathrm{L},\mathrm{M}) \subseteq \Upsilon_1 \times \Upsilon_2$. Then (L,M) is called ${\xi^*}_{\mathfrak{J}}$-pre-open set if $(\mathrm{U},\mathrm{V})\backslash \mathrm{Cl}_{\xi}(\mathrm{L},\mathrm{M}) \in \mathfrak{J}$ whenever,$(\mathrm{L},\mathrm{M}) \subseteq (\mathrm{U},\mathrm{V})$, where (U,V) is ξ-open set. Similarly(L,M) is called ${\xi^*}_{\mathfrak{J}}$-β-open set if $(\mathrm{U},\mathrm{V})\backslash \mathrm{I}_{\xi}(\mathrm{Cl}_{\xi}(\mathrm{L},\mathrm{M})) \in \mathfrak{J}$ whenever,$(\mathrm{L},\mathrm{M}) \subseteq (\mathrm{U},\mathrm{V})$, where (U,V) is ξ-open set.

Definition 2.11: If $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$and $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})is\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$. Then the mapping $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as $\xi_{\mathfrak{J}}$-semi ($\xi_{\mathfrak{J}}$-α) continuous map if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}_{\mathfrak{J}}$-semi ($\mathcal{T}_{\mathfrak{J}}$-α) open in $(\mathrm{Z},\mathcal{T})$ ∀ ξ-open sets $(\mathrm{L},\mathrm{M}) \in (\Upsilon_1,\Upsilon_2,\xi)$.

Definition 2.12: If $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$ and $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ is $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$. Then the mapping $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as strongly $\xi_{\mathfrak{J}}$-semi (strongly $\xi_{\mathfrak{J}}$-α) continuous map if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}_{\mathfrak{J}}$-semi ($\mathcal{T}_{\mathfrak{J}}$-α) clopen in $(\mathrm{Z},\mathcal{T})$ ∀ ξ-set $(\mathrm{L},\mathrm{M}) \in (\Upsilon_1,\Upsilon_2,\xi)$.

Definition 2.13: If $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}\mathrm{S}$ and $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ is $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$. Then the mapping $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as $\xi_{\mathfrak{J}}$-pre ($\xi_{\mathfrak{J}}$-β) continuous map if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}_{\mathfrak{J}}$-pre ($\mathcal{T}_{\mathfrak{J}}$-β) open in $(\mathrm{Z},\mathcal{T})$ ∀ ξ-open sets $(\mathrm{L},\mathrm{M}) \in (\Upsilon_1,\Upsilon_2,\xi)$.

Definition 2.14: If $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}\mathrm{S}$ and $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ is $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$. Then the mapping $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as strongly $\xi_{\mathfrak{J}}$-pre (strongly$\xi_{\mathfrak{J}}$-β) continuous map if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}_{\mathfrak{J}}$-pre ($\mathcal{T}_{\mathfrak{J}}$-β) clopen in $(\mathrm{Z},\mathcal{T})$ ∀ ξ-set $(\mathrm{L},\mathrm{M}) \in (\Upsilon_1,\Upsilon_2,\xi)$.

3. STRONGLY ${\xi^*}_{\mathfrak{J}}$-SEMI-CONTINUOUS MAPS

In this section, we established the relationship between strongly ${\xi^*}_{\mathfrak{J}}$-semi-continuous maps in $\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$and some other maps. The results have been shown by making the use of some counter examples.

Definition 3.1: If$(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}\mathrm{S}$ and $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{J})$ is$\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$. Then the mapping $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as **${\xi^*}_{\mathfrak{J}}$-semi-continuous map** if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is ${\mathcal{T}^*}_{\mathfrak{J}}$-semi-open in $(\mathrm{Z},\mathcal{T})$ ∀ ξ-open set $(\mathrm{L},\mathrm{M}) \in (\Upsilon_1,\Upsilon_2,\xi)$.

Definition 3.2: If $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T S}$ and $(\Upsilon_1,\Upsilon_2,\xi,\mathfrak{I})$ is$\mathrm{I}\xi_{\mathrm{T}}\mathrm{S}$. Then the mapping $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is known as **strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map** if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}^*_{\mathfrak{I}}$-semi-clopen in $(\mathrm{Z},\mathcal{T})$ $\forall$ ξ-sets$(\mathrm{L},\mathrm{M})\in(\Upsilon_1,\Upsilon_2,\xi)$.

Example 3.1: Let $\mathrm{Z}=\{1,2,3\}$,$\Upsilon_1=\{a_1,a_2\}$ and $\Upsilon_2=\{b_1,b_2\}$. Then clearly$\mathcal{T}=\{\emptyset,\{1,2\},\{2,3\},\mathrm{Z}\}$is $\mathrm{G_T}$, $\mathfrak{I}=\{\emptyset,\{2,3\},\mathrm{Z}\}$is an ideal on Z and $\xi=\{(\emptyset,\emptyset),(\{a_1\},\{b_1\}),(\{a_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$ is ξ_{T} from Υ_1 to Υ_2. Consider$\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ defined as$\mathcal{F}(2)=(a_1,b_1)=\mathcal{F}(3)$and $\mathcal{F}(1)=(a_2,\Upsilon_2)$. Therefore$\mathcal{F}^{-1}(\emptyset,\emptyset)=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\},\{b_1\})=\{2,3\}$, $\mathcal{F}^{-1}(\{a_1\},\{\Upsilon_2\})=\{\emptyset\}$, $f^{-1}(\{a_2\},\{\Upsilon_2\})=\{1\}$,$\mathcal{F}^{-1}(\{\emptyset\},\{b_1\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{b_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{\Upsilon_2\})=\{\emptyset\}$,$\mathcal{F}^{-1}(\{a_1\},\{\emptyset\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\},\{b_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\},\emptyset)=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\},\{b_1\})=\{\emptyset\}$,$\mathcal{F}^{-1}(\{a_2\},\{b_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{\emptyset\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_1\})=\{\emptyset\}\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_2\})=\{\emptyset\}$and $\mathcal{F}^{-1}(\Upsilon_1,\Upsilon_2)=\{\mathrm{Z}\}$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}^*_{\mathfrak{I}}$-semi-clopen set in $(\mathrm{Z},\mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map.

Remark 3.1: ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map ⇒⇍ strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map

Proof: Quite easy while the converse can be illustrated as follows.

Example 3.2: If $\mathrm{Z}=\{1,2,3\}$,$\Upsilon_1=\{a_1,a_2\}$ and $\Upsilon_2=\{b_1,b_2\}$. Then clearly $\mathcal{T}=\{\emptyset,\{1,3\},\{2,3\},\mathrm{Z}\}is\mathrm{G_T}$, $\mathfrak{I}=\{\emptyset,\{3\},\{2,3\}\}$ is an ideal on Z and $\xi=\{(\emptyset,\emptyset),(\{a_1\},\{b_2\}),(\{a_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$ is ξ_{T} from Υ_1 to Υ_2. Consider$\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ defined as $\mathcal{F}(1)=(a_1,b_1)$and$\mathcal{F}(2)=(a_2,\Upsilon_2)=\mathcal{F}(3)$. Therefore$\mathcal{F}^{-1}(\emptyset,\emptyset)=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\},\{b_1\})=\{1\}$, $\mathcal{F}^{-1}(\{a_1\},\{\Upsilon_2\})=\{\emptyset\}$, $f^{-1}(\{a_2\},\{\Upsilon_2\})=\{2,3\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{b_1\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{b_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{\Upsilon_2\})=\{\emptyset\}$,$\mathcal{F}^{-1}(\{a_1\},\{\emptyset\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\},\{b_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\},\emptyset)=\{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\},\{b_1\})=\{\emptyset\}$,$\mathcal{F}^{-1}(\{a_2\},\{b_2\})=\{2,3\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{\emptyset\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_1\})=\{2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_2\})=\{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\Upsilon_2)=\{\mathrm{Z}\}$. Hence we see inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) clopen set in $(\mathrm{Z},\mathcal{T})$. The map $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map but not ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map.

Proposition 3.1: Strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map ⇒⇍ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map

Proof: Suppose(L,M) beξ-set and$\mathcal{F}$ is strongly$\xi^*_{\mathfrak{I}}$-semi-continuous map. Therefore $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}^*_{\mathfrak{I}}$-semi-clopen set in $(\mathrm{Z},\mathcal{T})$. Since every $\mathcal{T}^*_{\mathfrak{I}}$-semi-clopen set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open, therefore $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open set in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

The converse can be illustrated as follows.

Example 3.3: If$\mathrm{Z}=\{1,2,3\}$,$\Upsilon_1=\{a_1,a_2\}$and $\Upsilon_2=\{b_1,b_2\}$. Then clearly $\mathcal{T}=\{\emptyset,\{1,2\},\{2,3\},\mathrm{Z}\}is\mathrm{G_T}$,$\mathfrak{I}=\{\emptyset,\{2,3\}\}$ is an ideal on Z and $\xi=\{(\emptyset,\emptyset),(\{a_1\},\{b_1\}),(\{a_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$ is ξ_{T} from $\Upsilon_1$to Υ_2. Consider $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ defined as $\mathcal{F}(1)=(a_1,b_1)=\mathcal{F}(3)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) open set in

$(\mathrm{Z}, \mathcal{T})$. Thus $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi-continuous map while not strongly$\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map because $\mathcal{F}^{-1}(\{a_1\},\{b_1\}) = \{1,3\}$ where $\{1,3\}$ is not $\mathcal{T}_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) clopen set in $(\mathrm{Z}, \mathcal{T})$. The map $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map but not strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map.

Definition 3.3: Suppose$(\mathrm{Z}, \mathcal{T})$ be $\mathrm{G_T S}$ and $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ be $\mathrm{I\xi_T S}$. Then the $\mathcal{F}: (\mathrm{Z}, \mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is known as **strongly $\xi^*_{\mathfrak{I}}$-α-continuous map** if $\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}^*_{\mathfrak{I}}$-α-clopen in $(\mathrm{Z}, \mathcal{T})$ $\forall$ ξ-open set $(\mathrm{L}, \mathrm{M}) \in (\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.4: The $\mathcal{F}$ is totally $\xi^*_{\mathfrak{I}}$-α-continuous map in Example 3.1

Remark 3.2: ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi^*_{\mathfrak{I}}$-α-continuous map.

Proof: Quite easy while the converse is illustrated as follows.

Proposition 3.2: Strongly $\xi^*_{\mathfrak{I}}$-α-continuous map $\Rightarrow\not\Leftarrow$ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

Proof: Suppose(L, M) be ξ-set and $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-α-continuous map. Therefore $\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}^*_{\mathfrak{I}}$-α-clopen set in $(\mathrm{Z}, \mathcal{T})$. Since every $\mathcal{T}^*_{\mathfrak{I}}$-α-clopen set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open, therefore $\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open set in $(\mathrm{Z}, \mathcal{T})$. Hence $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

The converse is illustrated in Example 3.3, where we found $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map but not totally $\xi^*_{\mathfrak{I}}$-α-continuous map.

Remark 3.3: Strongly ξ^*_{I}-semi-continuous map $\Rightarrow\not\Leftarrow$ strongly ξ^*_{I}-α-continuous map

Proof: Quite easy while the converse is illustrated as follows.

Example 3.5: Let $\mathrm{Z} = \{1, 2, 3,4\}$,$\Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \mathrm{Z}\}$ is$\mathrm{G_T}$, $\mathfrak{I} = \{\emptyset, \{1\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)$ is ξ_{T} from $\Upsilon_1$to Υ_2. Consider$\mathcal{F}: (\mathrm{Z}, \mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(2) = \mathcal{F}(3) = (a_1, b_1) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}^*_{\mathfrak{I}}$-α-clopen set but not $\mathcal{T}^*_{\mathfrak{I}}$-semi-clopen set in $(\mathrm{Z}, \mathcal{T})$. Thus$\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-α-continuous map but not strongly $\xi^*_{\mathfrak{I}}$-semi-continuous map.

Definition 3.4: Suppose$(\mathrm{Z}, \mathcal{T})$ be $\mathrm{G_T S}$ and $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ be$\mathrm{I\xi_T S}$. Then the mapping $\mathcal{F}: (\mathrm{Z}, \mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is known as **$\xi^*_{\mathfrak{I}}$-pre-continuous map** if $\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}^*_{\mathfrak{I}}$-pre-open in $(\mathrm{Z}, \mathcal{T})$ $\forall$ ξ-open set $(\mathrm{L}, \mathrm{M}) \in (\Upsilon_1, \Upsilon_2, \xi)$.

Definition 3.5: Suppose$(\mathrm{Z}, \mathcal{T})$ be $\mathrm{G_T S}$ and $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})be\mathrm{I\xi_T S}$. Then the mapping $\mathcal{F}: (\mathrm{Z}, \mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is known as **strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map** if $\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}^*_{\mathfrak{I}}$-pre-clopen in $(\mathrm{Z}, \mathcal{T})$ $\forall$ ξ-set $(\mathrm{L}, \mathrm{M}) \in (\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.6: Let $Z = \{1, 2, 3, \}, \Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, Z\}$ is G_T, $\mathfrak{I} = \{\emptyset, \{2\}, \{1,2\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_2\}, \{b_2\}), (\{a_1\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined by $\mathcal{F}(1) = (a_2, b_2)$ and $\mathcal{F}(2) = (a_1, \Upsilon_2) = \mathcal{F}(3)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{\Upsilon_2\}) = \{2,3\}$, $f^{-1}(\{a_2\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{\Upsilon_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{a_2\}, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{b_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = \{Z\}$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}^*_{\mathfrak{I}}$-pre-clopen set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map.

Remark 3.4: ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map ⇒⇍ strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map

Proof: Quite easy while converse is illustrated as follows.

Example 3.7: The map $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map set in Example 3.6 but not ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map.

Proposition 3.3: Strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map ⇒⇍ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map

Proof: Suppose (L, M) be ξ-set and $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}^*_{\mathfrak{I}}$-pre-clopen set in $(Z, \mathcal{T})$. Since every $\mathcal{T}^*_{\mathfrak{I}}$-pre-clopen set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open set in $(Z, \mathcal{T})$. Hence $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

The **c**onverse is illustrated as follows.

Example 3.8: Let $Z = \{1, 2, 3,4\}, \Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 3\}, \{1, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, Z\}$ is G_T, $\mathfrak{I} = \{\emptyset, \{2,3\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider the mapping $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(2) = (a_1, b_1) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$- α) open set but not $\mathcal{T}^*_{\mathfrak{I}}$-pre-open set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map but not strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map.

Definition 3.6: Suppose $(Z, \mathcal{T})$ be G_TS and $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ be $I\xi_TS$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as **strongly $\xi^*_{\mathfrak{I}}$-β-continuous map** if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}^*_{\mathfrak{I}}$-β-clopen in $(Z, \mathcal{T})$ ∀ ξ-set $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.9: The map $\mathcal{F}$ is strongly $\xi^*_{\mathfrak{I}}$-β-continuous map in Example 3.6.

Remark 3.5: ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map ⇒⇍ strongly $\xi^*_{\mathfrak{I}}$-β-continuous map

Proof: Quite easy while converse is illustrated as follows.

Example 3.10: The map $\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{J}}$-β-continuous map set in Example 3.6 but not ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map.

Proposition 3.4: Strongly $\xi^*{}_{\mathfrak{J}}$-β-continuous map ⇒⇍ $\xi_{\mathfrak{J}}$-semi ($\xi_{\mathfrak{J}}$-α) continuous map.

Proof: Suppose(L, M) be ξ-set and $\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{J}}$-β-continuous map. Therefore$\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}^*{}_{\mathfrak{J}}$-β-clopen set in $(\mathrm{Z}, \mathcal{T})$. Since every $\mathcal{T}^*{}_{\mathfrak{J}}$-β-clopen set is $\mathcal{T}_{\mathfrak{J}}$-semi ($\mathcal{T}_{\mathfrak{J}}$-α) open, therefore $\mathcal{F}^{-1}(\mathrm{L}, \mathrm{M})$ is $\mathcal{T}_{\mathfrak{J}}$-semi ($\mathcal{T}_{\mathfrak{J}}$-α) open set in $(\mathrm{Z}, \mathcal{T})$. Hence $\mathcal{F}$ is $\xi_{\mathfrak{J}}$-semi ($\xi_{\mathfrak{J}}$-α) continuous map.

The converse is illustrated as follows.

Example 3.11: The map$\mathcal{F}$ is $\xi_{\mathfrak{J}}$-semi ($\xi_{\mathfrak{J}}$-α) continuous map in Example 3.8 but not strongly $\xi^*{}_{\mathfrak{J}}$-β-continuous map.

Remark 3.6: Strongly $\xi^*{}_{\mathfrak{J}}$-continuous maps ⇎ strongly $\xi^*{}_{\mathfrak{J}}$-semi-continuous maps

This can be illustrated as follows.

Example 3.12: Let $\mathrm{Z} = \{1, 2, 3,4\}$,$\Upsilon_1 = \{\mathrm{a}_1, \mathrm{a}_2\}$ and $\Upsilon_2 = \{\mathrm{b}_1, \mathrm{b}_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \mathrm{Z}\}$is $\mathrm{G_T}$, $\mathfrak{J} = \{\emptyset, \{1\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{\mathrm{a}_2\}, \{\mathrm{b}_2\}), (\{\mathrm{a}_1\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is $\xi_{\mathrm{T}}$from Υ_1 to Υ_2. Consider$\mathcal{F}: (\mathrm{Z}, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (\mathrm{a}_1, \Upsilon_2)$and $\mathcal{F}(2) = \mathcal{F}(3) = (\mathrm{a}_2, \mathrm{b}_2) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}^*{}_{\mathfrak{J}}$-clopen set but not $\mathcal{T}^*{}_{\mathfrak{J}}$-semi-clopen set in $(\mathrm{Z}, \mathcal{T})$. Thus$\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{J}}$-continuous map but not strongly $\xi^*{}_{\mathfrak{J}}$-semi-continuous map.

Example 3.13: Let $\mathrm{Z} = \{1, 2, 3\}$,$\Upsilon_1 = \{\mathrm{a}_1, \mathrm{a}_2\}$ and $\Upsilon_2 = \{\mathrm{b}_1, \mathrm{b}_2\}$. Then $\mathcal{T} = \{\emptyset, \{1, 3\}, \{2, 3\}, \mathrm{Z}\}$is $\mathrm{G_T}$, $\mathfrak{J} = \{\emptyset, \{1, 3\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{\mathrm{a}_2\}, \{\mathrm{b}_2\}), (\{\mathrm{a}_1\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is $\xi_{\mathrm{T}}$from Υ_1 to Υ_2. Consider$\mathcal{F}: (\mathrm{Z}, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (\mathrm{a}_1, \Upsilon_2)$ and $\mathcal{F}(2) = (\mathrm{a}_2, \mathrm{b}_2) = \mathcal{F}(3)$. Hence the inverse image of every ξ-set is $\mathcal{T}^*{}_{\mathfrak{J}}$-semi-clopen set in $(\mathrm{Z}, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{J}}$-semi-continuous map but not strongly $\xi^*{}_{\mathfrak{J}}$-continuous map.

Remark 3.7: Strongly $\xi^*{}_{\mathfrak{J}}$-continuous maps ⇎ strongly $\xi^*{}_{\mathfrak{J}}$-α-continuous maps

This is illustrated as follows.

Example 3.14: The map$\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{J}}$-continuous map in Example 3.12, but not strongly $\xi^*{}_{\mathfrak{J}}$-α-continuous map.

Example 3.15: The map $\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{J}}$-α-continuous map in Example 3.13 but not strongly $\xi^*{}_{\mathfrak{J}}$-continuous map.

Remark 3.8: Strongly $\xi^*{}_{\mathfrak{J}}$-semi-continuous maps ⇎ strongly $\xi^*{}_{\mathfrak{J}}$-pre-continuous maps

This is illustrated as follows.

Example 3.16: Let $Z = \{1, 2, 3, 4\}, \Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, Z\}$is G_T, $\mathfrak{I} = \{\emptyset, \{1,2,4\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_2\}, \{b_2\}), (\{a_1\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is $\xi_T$from Υ_1 to Υ_2. Consider$\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (a_1, \Upsilon_2) = \mathcal{F}(3)$and $\mathcal{F}(2) = (a_2, b_2) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-clopen set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-pre-clopen set in $(Z, \mathcal{T})$. Thus$\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{I}}$-semi-continuous map but not strongly $\xi^*{}_{\mathfrak{I}}$-pre-continuous map.

Example 3.17: Let $I = \{\emptyset, \{3\}\}$ on ideal on Z and $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (a_2, b_2) = \mathcal{F}(2)$ and $\mathcal{F}(3) = (a_1, \Upsilon_2) = \mathcal{F}(4)$ in Example 5.16. Hence we see that the inverse image of every ξ-set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}^*{}_{\mathfrak{I}}$-pre-clopen set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-clopen set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{I}}$-pre-continuous map but not strongly $\xi^*{}_{\mathfrak{I}}$-semi-continuous map.

Remark 3.13: Strongly $\xi^*{}_{\mathfrak{I}}$-continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi^*{}_{\mathfrak{I}}$-pre ($\xi^*{}_{\mathfrak{I}}$-β) continuous map

Proof: Quite easy while the converse is illustrated as follows.

Example 3.18: Let $Z = \{1, 2, 3, 4\}, \Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 3\}, \{1, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, Z\}$is G_T, $\mathfrak{I} = \{\emptyset, \{2\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined by $\mathcal{F}(1) = (a_1, b_1)$and $\mathcal{F}(2) = \mathcal{F}(3) = \mathcal{F}(4) = (a_2, \Upsilon_2)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-pre ($\mathcal{T}_{\mathfrak{I}}$-β) clopen set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-clopen set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi^*{}_{\mathfrak{I}}$-pre ($\xi^*{}_{\mathfrak{I}}$-β) continuous map but not strongly $\xi^*{}_{\mathfrak{I}}$-continuous map.

4. CONCLUSION

We introduced and study useful concept of strongly $\xi^*{}_{\mathfrak{I}}$-semi-continuous and strongly $\xi^*{}_{\mathfrak{I}}$-pre-continuous maps and established the relationship between above discuses maps and several other maps like ξ, ξ-semi, ξ-pre, ξ-α, ξ-β, $\xi_{\mathfrak{I}}$-semi, $\xi_{\mathfrak{I}}$-pre, $\xi_{\mathfrak{I}}$-α, $\xi_{\mathfrak{I}}$-β and $\xi^*{}_{\mathfrak{I}}$-continuous maps etc. The significant of results have been shown by several counter examples. In the present direction, we have categorised maps in generalized binary ideal topological spaces and investigated the behaviour of presented maps by utilizing ideal. We conclude the results in this research paper given below:

ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi^*{}_{\mathfrak{I}}$-semi-continuous map

Strongly $\xi^*{}_{\mathfrak{I}}$-semi-continuous map $\Rightarrow\not\Leftarrow$ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map

ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi^*{}_{\mathfrak{I}}$-α-continuous map.

Strongly $\xi^*{}_{\mathfrak{I}}$-α-continuous map $\Rightarrow\not\Leftarrow$ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

Strongly ξ^*_I-semi-continuous map $\Rightarrow\nLeftarrow$ strongly ξ^*_I-α-continuous map

ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\nLeftarrow$ strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map

Strongly $\xi^*_{\mathfrak{I}}$-pre-continuous map $\Rightarrow\nLeftarrow$ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map

ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\nLeftarrow$ strongly $\xi^*_{\mathfrak{I}}$-β-continuous map

Strongly $\xi^*_{\mathfrak{I}}$-β-continuous map $\Rightarrow\nLeftarrow$ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

Strongly $\xi^*_{\mathfrak{I}}$-continuous maps $\nLeftrightarrow$ strongly $\xi^*_{\mathfrak{I}}$-semi-continuous maps

Strongly $\xi^*_{\mathfrak{I}}$-continuous maps $\nLeftrightarrow$ strongly $\xi^*_{\mathfrak{I}}$-α-continuous maps

Strongly $\xi^*_{\mathfrak{I}}$-semi-continuous maps $\nLeftrightarrow$ strongly $\xi^*_{\mathfrak{I}}$-pre-continuous maps

Strongly $\xi^*_{\mathfrak{I}}$-continuous map $\Rightarrow\nLeftarrow$ strongly $\xi^*_{\mathfrak{I}}$-pre ($\xi^*_{\mathfrak{I}}$-β) continuous map

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