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OPTIMIZING TWO-WAREHOUSE RED WINE INVENTORY MANAGEMENT WITH DETERIORATION: A LIFO DISPATCH STRATEGY

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Abstract

In this study, a two-warehouse inventory model for the red wine industry is presented, focusing on items that deteriorate. The model takes into account the effects of inflation and operates on the LIFO (Last In, First Out) distribution policy. The goal is to minimize the total inventory cost, including the cost of maintaining inventory and the cost of purchasing new inventory. The model assumes a constant and known demand for red wine over time, with no permitted shortages. The stock system consists of warehouse A and warehouse B, with starting stocks fixed in each case. Inventory balance equations track inventory levels at each warehouse, taking into account transfers between warehouses and customer demand. The LIFO shipping policy is implemented to determine the quantities transferred between warehouses. Inflation is factored into the model by adjusting the cost of buying new inventory. This adjustment may be based on historical inflation rates or other relevant factors. The objective function is to minimize the total inventory cost, taking into account inventory costs and inflation-adjusted purchasing costs. The model provides a framework for making decisions about stock levels and transfer rates between warehouses. By optimizing these decisions, the model aims to achieve cost savings while ensuring that demand is met without bottlenecks. The model results can serve as a guide for inventory management strategies in the red wine industry, taking into account the specific challenges presented by the spoiled nature of the product and the impact of inflation.

Keywords:- Inventory, owned warehouse, rented warehouse, ramp type demand, deteriorating items, inflation, without Shortages and LIFO dispatching policy.

1. Introduction

The red wine industry faces unique inventory management challenges due to the nature of its product, which is a spoiling item. Over time, the quality and value of red wine declines, making it crucial for businesses to effectively manage their inventory. Additionally, inflation can have a significant impact on the cost of purchasing new inventory, further complicating the decision-making process. In this study, we propose a two-storage stock model specifically designed for the red wine industry. The model takes into account the deteriorated nature of red wine and inflation and is based on the LIFO (Last In, First Out) shipping policy. The LIFO policy assumes that inventory purchased last is shipped first, reflecting typical industry storage and retrieval practices. The goal of the inventory model is to minimize the overall cost of inventory while ensuring that customer demand is met without shortages. By optimizing inventory levels and transfer rates between warehouses, companies can realize cost savings and avoid overstocking or understocking. This results in improved profitability and customer satisfaction. To capture the effects of spoilage, the model tracks inventory levels in warehouse A and warehouse B over time. The equilibrium equations take into account customer demand, transfers between warehouses and initial inventory levels. The model dynamically adjusts stock levels based on these factors, while taking into account deterioration in the condition of red wine. Inflation is another critical factor considered in the model. The cost of purchasing new inventory is adjusted for the impact of inflation. This adjustment can be based on historical inflation rates or other relevant factors to ensure that the model provides a realistic representation of industry cost dynamics. Overall, the proposed inventory model provides a comprehensive framework for decision making in the red wine industry. By addressing the unique challenges of item spoilage, inflation, and LIFO shipping policy, businesses can optimize their inventory management strategies and increase their competitive advantage. The following sections of this study detail the details of the model, its formulation, and the potential benefits it offers to the red wine industry.

2. Assumptions and Notations:

In developing the mathematical model of the Red wine industry inventory system the following assumptions are being made:

1. A single item is considered over a prescribed
2. period T units of time.
3. The demand rate D(t) at time t is deterministic and taken as a ramp type function of time i.e. $D(t)=\alpha_0 e^{-\alpha_2\{t-\{t-(t_\alpha+t_L)\}H[t-(t_\alpha+t_L)]\}}$, $\alpha_0>0,\ \alpha_2>0$, where $H\left[t-\left(t_\alpha+t_L\right)\right]$ is the Heaviside's function defined as $H\left[t-\left(t_\alpha+t_L\right)\right]=\begin{cases}0, & t<\left(t_\alpha+t_L\right)\\ 1, & t\geq\left(t_\alpha+t_L\right)\end{cases}$
4. The replenishment rate is infinite and lead-time is zero.
5. When the demand for goods is more than the supply. Shortages will occur. Customers encountering shortages will either wait for the vender to reorder (backlogging cost involved) or go to other vendors (lost sales cost involved). In this model shortages are allowed and the backlogging rate is $exp(-\alpha_3 t)$, when Red wine industry inventory is in shortage. The backlogging parameter α_3 is a positive constant.
6. The variable rate of deterioration in both warehouse is taken as $\alpha_1(t) = \alpha_1 t$. Where $0<\alpha_1<<1$ and only applied to on hand Red wine industry inventory.
7. No replacement or repair of deteriorated items is made during a given cycle.

8. The Red wine industry owned warehouse (OW) has a fixed capacity of W units; the Red wine industry rented warehouse (RW) has unlimited capacity.
9. The goods of OW are consumed only after consuming the goods kept in RW.

In addition, the following notations are used throughout this paper:

$\Pi_{ow}^{lifo}(t)$	The Red wine industry inventory level in OW at any time t.
$\Pi_{rw}^{lifo}(t)$	The Red wine industry inventory level in RW at any time t.
α_w	The capacity of the own warehouse.
Q	The ordering quantity per cycle.
T	Planning horizon.
α_4	Inflation rate.
α_{hcow}	The holding cost per unit per unit time in Red wine industry OW.
α_{hcrw}	The holding cost per unit per unit time in Red wine industry RW.
α_{dc}	The deterioration cost per unit.
α_{sc}	The shortage cost per unit per unit time.
α_{opc}	The opportunity cost due to lost sales.
α_{OC}	The replenishment cost per order.

3. Formulation and Solution of the Model

The Red wine industry inventory levels at OW are governed by the following differential equations under LIFO dispatching policy:

$$\frac{d\Pi_{ow}^{lifo}(t)}{dt} = \left[-\alpha_1(t) I^{lifo}(t)\right] \qquad 0 \le t < (t_\alpha + t_L) \qquad (1)$$

$$\frac{d\Pi_{ow}^{lifo}(t)}{dt} + \alpha_1(t) I^{lifo}(t) = -\alpha_0 e^{-\alpha_2 (t_\alpha + t_L)}, \qquad (t_\alpha + t_L) \le t \le (t_1 + t_L) \qquad (2)$$

And

$$\left[\frac{d\Pi_{ow}^{lifo}(t)}{dt}\right]=\left[-\alpha_0 e^{-\alpha_2(t_\alpha+t_L)}e^{-\alpha_3 t}\right] \qquad (t_1+t_L)\le t\le(T+t_L) \qquad (3)$$

with the boundary conditions,

$$\Pi_{ow}^{lifo}(0)=\alpha_w \text{ and } I^{lifo}(t_1+t_L)=0 \qquad (4)$$

The solutions of equations (1), (2) and (3) are given by

$$\Pi_{ow}^{lifo}(t)=\alpha_w e^{-\alpha_1 t^2/2}, \qquad 0\le t<(t_\alpha+t_L) \qquad (5)$$

$$\Pi_{ow}^{lifo}(t)=\left[\alpha_0 e^{-\alpha_2(t_\alpha+t_L)}\left\{\begin{matrix}\{(t_1+t_L)-t\}+\\ \frac{\alpha_1(t_1+t_L)^3-t^3)}{6}\end{matrix}\right\}e^{-\alpha_1 t^2/2}\right], \quad (t_\alpha+t_L)\le t\le(t_1+t_L) \quad (6)$$

And $$\Pi_{ow}^{lifo}(t)=\left[\frac{\alpha_0}{\alpha_3}e^{-\alpha_2(t_\alpha+t_L)}\left\{e^{-\alpha_3 t}-e^{-\alpha_3(t_1+t_L)}\right\}\right] \quad (t_1+t_L)\le t\le(T+t_L) \quad (7)$$

respectively.

The Red wine industry inventory level at RW is governed by the following differential equations:

$$\frac{d\Pi_{rw}^{lifo}(t)}{dt}+\alpha_1(t)I^{lifo}(t)=-\alpha_0 e^{-\alpha_2 t}, \qquad 0\le t<(t_\alpha+t_L) \qquad (8)$$

With the boundary condition $\Pi_{rw}^{lifo}(0)=0$ the solution of the equation (8) is

$$\Pi_{rw}^{lifo}(t)=\left[\alpha_0\left\{\begin{matrix}\{(t_\alpha+t_L)-t\}-\\ \frac{\alpha_2}{2}\left((t_\alpha+t_L)^2-t^2\right)+\\ \frac{\alpha_1}{6}((t_1+t_L)^3-t^3)\end{matrix}\right\}e^{-\alpha_1 t^2/2}\right] \qquad (t_\alpha+t_L)\le t\le(t_1+t_L) \qquad (9)$$

Due to continuity of $\Pi_{ow}^{lifo}(t)$ at point $t=(t_\alpha+t_L)$ it follows from equations (5) and (6), one has

$$\alpha_w e^{-\alpha_1(t_\alpha+t_L)^2/2}=\left[\alpha_0 e^{-\alpha_2(t_\alpha+t_L)}\left\{\begin{matrix}\{(t_1+t_L)-(t_\alpha+t_L)\}+\\ \frac{\alpha_1\left\{(t_1+t_L)^3-(t_\alpha+t_L)^3\right\}}{6}\end{matrix}\right\}e^{-\alpha_1(t_\alpha+t_L)^2/2}\right]$$

$$\alpha_w = \left[\alpha_0 e^{-\alpha_2 (t_\alpha + t_L)} \left\{ \begin{array}{l} \left\{ (t_1 + t_L) - (t_\alpha + t_L) \right\} + \\ \dfrac{\alpha_1 \left\{ (t_1 + t_L)^3 - (t_\alpha + t_L)^3 \right\}}{6} \end{array} \right\} \right] \tag{10}$$

The total average cost consists of following elements:

(i) Ordering cost per cycle = α_{OC} (11)

(ii) Holding cost per cycle (C_{HO}) in OW

$$C_{HO} = \left[\alpha_{hcow} \left\{ \begin{array}{l} \displaystyle\int_0^{(t_\alpha + t_L)} \Pi_{ow}^{lifo}(t) e^{-\alpha_4 t} dt + \\ \displaystyle\int_{(t_\alpha + t_L)}^{(t_1 + t_L)} \Pi_{ow}^{lifo}(t) e^{-\alpha_4 \left\{ (t_\alpha + t_L) + t \right\}} dt \end{array} \right\} \right]$$

$$C_{HO} = \left[\alpha_{hcow} \left\{ \begin{array}{l} \alpha_w \left\{ \left((t_\alpha + t_L) - \dfrac{\alpha_4 (t_\alpha + t_L)^2}{2} - \dfrac{\alpha_1 (t_\alpha + t_L)^3}{6} \right) \right\} + \\ \alpha_0 e^{-\alpha_2 ((t_\alpha + t_L) + \alpha_4)} \left\{ \begin{array}{l} \dfrac{(t_1 + t_L)^2}{2} - \dfrac{\alpha_4 (t_1 + t_L)^3}{6} + \\ \dfrac{\alpha_1 (t_1 + t_L)^4}{12} - \dfrac{\alpha_4 \alpha_1}{20} (t_1 + t_L)^5 - \\ \dfrac{(t_\alpha + t_L)}{2} \left(2(t_1 + t_L) - (t_\alpha + t_L) \right) - \\ \dfrac{\alpha_1 (t_\alpha + t_L)}{24} \left(4(t_1 + t_L)^3 - (t_\alpha + t_L)^3 \right) + \\ \dfrac{\alpha_4 (t_\alpha + t_L)^2}{6} \left(3(t_1 + t_L) - 2(t_\alpha + t_L) \right) + \\ \dfrac{\alpha_4 \alpha_1 (t_\alpha + t_L)^2}{30} \left(5(t_1 + t_L)^3 - 3(t_\alpha + t_L)^3 \right) + \\ \dfrac{\alpha_1 (t_\alpha + t_L)^3}{24} \left(4(t_1 + t_L) - 3(t_\alpha + t_L) \right) \end{array} \right\} \end{array} \right\} \right] \tag{12}$$

(iii) Holding cost per cycle (C_{HR}) in RW

$$C_{HR} = \left[\alpha_{hcrw} \left\{ \int_{0}^{(t_\alpha + t_L)} \Pi_{rw}^{lifo}(t) e^{-\alpha_4 t} dt \right\} \right]$$

$$C_{HR} = \left[\alpha_{hcrw} \alpha_0 \left\{ \begin{array}{l} \frac{(t_\alpha + t_L)^2}{2} - \frac{(3\alpha_2 + \alpha_4)}{6} (t_\alpha + t_L)^3 + \\ \left(\frac{\alpha_1}{12} + \frac{\alpha_2 \alpha_4}{8} \right) (t_\alpha + t_L)^4 - \\ \left(\frac{\alpha_4 \alpha_1}{20} - \frac{\alpha_2 \alpha_1}{30} \right) (t_\alpha + t_L)^5 \end{array} \right\} \right] \qquad \ldots (3.13)$$

(iv) Cost of deteriorated units per cycle (C_D)

$$= \left[\alpha_{dc} \left\{ \begin{array}{l} \int_{0}^{(t_\alpha + t_L)} \alpha_1 t \Pi_{rw}^{lifo}(t) e^{-\alpha_4 t} dt + \\ \int_{0}^{(t_\alpha + t_L)} \alpha_1 t \Pi_{ow}^{lifo}(t) e^{-\alpha_4 t} dt + \\ \int_{(t_\alpha + t_L)}^{(t_1 + t_L)} \alpha_1 t \Pi_{ow}^{lifo}(t) e^{-\alpha_4 \{ t + (t_\alpha + t_L) \}} dt \end{array} \right\} \right] \qquad (14)$$

(v) Shortage cost per cycle (C_S)

$$= \alpha_{sc} \left[\int_{(t_1 + t_L)}^{(T + t_L)} -\Pi_{ow}^{lifo}(t) e^{-\alpha_4 \{ (t_1 + t_L) + t \}} dt \right]$$

$$= \frac{-\alpha_0 \alpha_{sc} e^{-[\alpha_4 (t_1 + t_L) + \alpha_2 (t_\alpha + t_L)]}}{\alpha_3} \left[\int_{(t_1 + t_L)}^{(T + t_L)} e^{-(\alpha_4 + \alpha_3) t} dt - e^{-\alpha_3 (t_1 + t_L)} \int_{(t_1 + t_L)}^{(T + t_L)} e^{-\alpha_4 t} dt \right]$$

$$= \left[\frac{\alpha_0 \alpha_{sc} e^{-(\alpha_4 (t_1 + t_L) + \alpha_2 (t_\alpha + t_L))}}{\alpha_3 \alpha_4 (\alpha_3 + \alpha_4)} \left\{ \begin{array}{l} \alpha_3 e^{-(\alpha_3 + \alpha_4)(t_1 + t_L)} + \\ e^{-\alpha_4 (T + t_L)} \left\{ \begin{array}{l} r e^{-\alpha_3 (T + t_L)} - \\ (\alpha_3 + \alpha_4) e^{-\alpha_3 (t_1 + t_L)} \end{array} \right\} \end{array} \right\} \right] \qquad (15)$$

(vi) Opportunity cost due to lost sales per cycle (C_0)

$$= \alpha_{opc} \int_{(t_1+t_L)}^{(T+t_L)} \alpha_0 (1 - e^{-\alpha_3 t}) e^{-\alpha_2 (t_\alpha + t_L)} e^{-\alpha_4 \{(t_1+t_L)+t\}} dt$$

$$= \left[\frac{\alpha_{opc} \alpha_0 e^{-\{\alpha_2 (t_\alpha + t_L) + \alpha_4 (t_1 + t_L)\}}}{\alpha_4 (\alpha_3 + \alpha_4)} \begin{Bmatrix} e^{-\alpha_4 (t_1+t_L)} \left\{ (\alpha_3 + \alpha_4) - \alpha_4 e^{-\alpha_3 (t_1+t_L)} \right\} - \\ e^{-\alpha_4 (T+t_L)} \left\{ (\alpha_3 + \alpha_4) - \alpha_4 e^{-\alpha_3 (T+t_L)} \right\} \end{Bmatrix} \right] \quad (16)$$

Therefore, the total average cost per unit time of our model is obtained as follows

$$TC \begin{pmatrix} t_1 + t_L, \\ T + t_L \end{pmatrix} = \frac{1}{(T+t_L)} \begin{bmatrix} \text{Ordering cost +Holding cost in OW} \\ \text{+ Holding cost in RW+ Deterioration cost} \\ \text{+ Shortage cost + Opportunity cost} \end{bmatrix} = \frac{R \begin{pmatrix} t_1 + t_L, \\ T + t_L \end{pmatrix}}{(T+t_L)} \quad (17)$$

Appendix A (18)

To minimize the total cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial (t_1 + t_L)} = 0 \quad (19)$$

and $$\frac{\partial TC}{\partial (T + t_L)} = 0 \quad (20)$$

provided, they satisfy the following conditions

and
$$\left. \begin{array}{l} \dfrac{\partial^2 TC}{\partial (t_1 + t_L)^2} > 0, \dfrac{\partial^2 TC}{\partial (T + t_L)^2} > 0 \\ \left(\dfrac{\partial^2 TC}{\partial (t_1 + t_L)^2} \right) \left(\dfrac{\partial^2 TC}{\partial (T + t_L)^2} \right) - \left(\dfrac{\partial^2 TC}{\partial (t_1 + t_L) \partial (T + t_L)} \right)^2 > 0 \end{array} \right\} \quad (21)$$

The equation (3.19) is equivalent to the following equation

$$\frac{\partial TC}{\partial(t_1+t_L)}=\left[\frac{\alpha_o}{(T+t_L)}\left[\begin{array}{l}\left\{e^{-\alpha_2\left(\begin{smallmatrix}(t_\alpha+t_L)\\+\alpha_4\end{smallmatrix}\right)}\left(\begin{array}{l}(t_1+t_L)-\frac{\alpha_4(t_1+t_L)^2}{2}+\\ \frac{\alpha_1(t_1+t_L)^3}{3}-\frac{\alpha_4\alpha_1(t_1+t_L)^4}{4}-\\ (t_\alpha+t_L)-3\alpha_1(t_\alpha+t_L)(t_1+t_L)^2+\\ \frac{\alpha_4(t_\alpha+t_L)^2}{2}+\frac{\alpha_4\alpha_1(t_\alpha+t_L)^2(t_1+t_L)^2}{2}+\\ \frac{\alpha_1(t_\alpha+t_L)^3}{6}\end{array}\right)\right\}+\\ \alpha_{dc}\alpha_1 e^{-\alpha_2\left(\begin{smallmatrix}(t_\alpha+t_L)\\+\alpha_4\end{smallmatrix}\right)}\left\{\left(\begin{array}{l}\frac{(t_1+t_L)^2}{2}-\frac{\alpha_4(t_1+t_L)^3}{3}+\\ \frac{\alpha_1(t_1+t_L)^4}{8}-\frac{\alpha_4\alpha_1(t_1+t_L)^5}{6}-\\ \frac{(t_\alpha+t_L)^2}{2}-\frac{\alpha_1(t_\alpha+t_L)^2(t_1+t_L)^2}{4}-\\ \frac{\alpha_4(t_\alpha+t_L)^3}{3}-\frac{\alpha_4\alpha_1(t_\alpha+t_L)^3(t_1+t_L)^2}{6}-\\ \frac{\alpha_1(t_\alpha+t_L)^4}{8}\end{array}\right)\right\}+\\ \frac{\alpha_{sc}e^{-\alpha_2(t_\alpha+t_L)}}{\alpha_3\alpha_4(\alpha_3+\alpha_4)}\left\{\left(\begin{array}{l}-\alpha_3(\alpha_3+2\alpha_4)e^{-(\alpha_3+2\alpha_4)(t_1+t_L)}-\\ {\alpha_4}^2e^{-\alpha_4(t_1+t_L)-(\alpha_4+\alpha_3)(T+t_L)}+\\ (\alpha_3+\alpha_4)^2\,e^{-(\alpha_4+\alpha_3)(t_1+t_L)-\alpha_4(T+t_L)}\end{array}\right)\right\}+\\ \frac{\alpha_{opc}e^{-\alpha_2(t_\alpha+t_L)}}{\alpha_4(\alpha_3+\alpha_4)}\left\{\left(\begin{array}{l}-2\alpha_4(\alpha_3+\alpha_4)e^{-2\alpha_4(t_1+t_L)}+\\ \alpha_4(\alpha_3+2\alpha_4)e^{-(\alpha_3+2\alpha_4)(t_1+t_L)}+\\ \alpha_4 e^{-\alpha_4(t_1+t_L)}\left(\begin{array}{l}(\alpha_3+\alpha_4)e^{-\alpha_4(T+t_L)}-\\ \alpha_4 e^{-(\alpha_3+\alpha_4)(T+t_L)}\end{array}\right)\end{array}\right)\right\}\end{array}\right]\right]=0 \quad (22)$$

Also equation (20) is equivalent to

$$\left[R-\frac{\alpha_o}{\alpha_3}\left\{\begin{matrix} e^{-\alpha_4(T+t_L)}- \\ e^{-(\alpha_4+\alpha_3)(T+t_L)}\end{matrix}\right\}e^{\left\{\begin{matrix}-\alpha_4(t_1+t_L) \\ +\alpha_2(t_\alpha+t_L)\end{matrix}\right\}}\left(\alpha_{sc}+\alpha_{opc}\alpha_3\right)\right]=0 \qquad (23)$$

Equations (22) and (23) are highly nonlinear equations. Therefore, numerical solution of these equations can be obtained by using the software **MATLAB 7.0.1**.

4. Particular Cases: Without shortages:

When $\alpha_3 \to \infty$ (i.e., the fraction of shortages backordered is zero), we get $\left(T+t_L\right)\approx 0$. The model reduces to the case where shortages are not allowed and hence the total average cost per unit time in equation (18) becomes:

$$TC\left(t_1+t_L\right)=\frac{1}{\left(T+t_L\right)}\left[\begin{matrix}\text{Ordering cost +Holding cost + Deterioration cost} \\ \text{+ Shortage cost + Opportunity cost}\end{matrix}\right]=\frac{R\left(t_1+t_L\right)}{\left(T+t_L\right)} \quad (24)$$

Appendix B (25)

The necessary condition to find the optimal solution of K (t_1) is

$$\frac{\partial TC}{\partial(t_1+t_L)}=\frac{\alpha_o}{(T+t_L)}\left[\begin{matrix} e^{-\alpha_2((t_\alpha+t_L)+\alpha_4)}\left(\begin{matrix}(t_1+t_L)-\frac{\alpha_4(t_1+t_L)^2}{2}+\frac{\alpha_1(t_1+t_L)^3}{3} \\ -\frac{\alpha_4\alpha_1(t_1+t_L)^4}{4}-(t_\alpha+t_L)-3\alpha_1(t_\alpha+t_L)(t_1+t_L)^2 \\ +\frac{\alpha_4(t_\alpha+t_L)^2}{2}+\frac{\alpha_4\alpha_1(t_\alpha+t_L)^2(t_1+t_L)^2}{2}+\frac{\alpha_1(t_\alpha+t_L)^3}{6}\end{matrix}\right) \\ +\alpha_{dc}\alpha_1 e^{-\alpha_2((t_\alpha+t_L)+\alpha_4)}\left(\begin{matrix}\frac{(t_1+t_L)^2}{2}-\frac{\alpha_4(t_1+t_L)^3}{3}+\frac{\alpha_1(t_1+t_L)^4}{8}-\frac{\alpha_4\alpha_1(t_1+t_L)^5}{6} \\ -\frac{(t_\alpha+t_L)^2}{2}-\frac{\theta(t_\alpha+t_L)^2(t_1+t_L)^2}{4}-\frac{\alpha_4(t_\alpha+t_L)^3}{3} \\ -\frac{\alpha_4\alpha_1(t_\alpha+t_L)^3(t_1+t_L)^2}{6}-\frac{\alpha_1(t_\alpha+t_L)^4}{8}\end{matrix}\right)\end{matrix}\right] \qquad (26)$$

$=0$

5. Numerical Illustration:

To illustrate the model numerically the following parameter values are considered.

$\alpha_1 = 0.0022\ unit, \alpha_0 = 502\ units, \alpha_2 = 0.22\ unit, \alpha_3 = 0.12\ unit, \alpha_4 = 0.052\ unit, t_\alpha = 0.2\ year$
$\alpha_{OC} = Rs.\ 1002\ per\ order, \alpha_{hcow} = Rs.\ 4.02\ per\ unit\ per\ year, \alpha_{hcrw} = Rs.\ 20.0\ per\ unit$
$T = 12\ year, \alpha_{sc} = Rs.\ 22.0\ per\ unit\ per\ year, \alpha_{opc} = Rs.\ 42.0\ per\ unit$

Then for the minimization of total average cost and with help of software. the optimal policy can be obtained such as: $t_1 = 0.599224$ year, S = 138.597235 units and TC = Rs.758.115354 per year.

6. Conclusion

The two-warehouse inventory model for the red wine industry provides a valuable framework for effective inventory management, taking into account item spoilage, inflation, and LIFO (Last In , First Out). By integrating these factors into the model, companies can make informed decisions about inventory levels and transfer rates between warehouses, reducing costs and improving customer satisfaction. Deteriorating red wine condition requires careful inventory management to avoid overstocking and product obsolescence. The proposed model takes this into account by dynamically adjusting stock levels based on equilibrium equations that take into account customer demand and transfers between stocks. By optimizing these inventories, companies can minimize storage costs and reduce spoilage losses. Inflation is another critical factor that can significantly affect the cost of purchasing new inventory. By adjusting purchase costs in the model for inflation, companies can make more accurate cost calculations and ensure that model results are consistent with current economic conditions. This allows them to make cost-effective decisions while maintaining desired service levels. The LIFO shipping policy reflects typical industry storage and retrieval practices. By adopting this policy in the model, companies can ensure that inventory purchased last is shipped first, which is consistent with industry practice and minimizes potential obsolescence costs. Overall, the dual warehouse inventory model presented in this study provides a comprehensive solution to inventory management challenges in the red wine industry. By taking into account the unique characteristics of red wine, inflation, and LIFO shipping policies, companies can optimize inventory levels, reduce costs, and improve overall operational efficiency. It should be noted that the efficiency of the model can be increased by incorporating additional factors such as batch ordering, lead times and storage capacity constraints depending on the specific needs of the red wine industry. Additionally, future research could explore the integration of advanced forecasting techniques and real-time data to improve the accuracy of demand forecasting and further improve model performance.

Ultimately, by adopting and implementing the two-warehouse inventory model, considering degradation, inflation, and LIFO shipping policy, companies in the red wine industry can gain a competitive advantage, reduce costs and provide superior customer service, contributing to their long-term success in the marketplace.

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Appendix A

$$
= \alpha_{dc}\alpha_1 \left\{ \begin{array}{l}
\left\{ \alpha_0 \left(\begin{array}{l}
\frac{1}{6}(t_\alpha + t_L)^3 - \\
\left(\frac{\alpha_2}{4} + \frac{\alpha_4}{12}\right)(t_\alpha + t_L)^4 + \\
\left(\frac{\alpha_1}{40} + \frac{\alpha_4\alpha_2}{15}\right)(t_\alpha + t_L)^5 - \\
\left(\frac{\alpha_4\alpha_1}{36} - \frac{\alpha_2\alpha_1}{24}\right)(t_\alpha + t_L)^6
\end{array} \right) \right\} + \\
\left\{ \alpha_w \left(\frac{(t_\alpha + t_L)^2}{2} - \frac{\alpha_4 (t_\alpha + t_L)^3}{3} \frac{\alpha_1 (t_\alpha + t_L)^4}{8} \right) \right\} + \\
\left\{ \alpha_0 e^{-(t_\alpha + t_L)(\alpha_2 + \alpha_4)} \left[\begin{array}{l}
\frac{(t_1 + t_L)^3}{6} - \frac{\alpha_4 (t_1 + t_L)^4}{12} + \\
\frac{\alpha_1 (t_1 + t_L)^5}{40} - \frac{\alpha_4\alpha_1 (t_1 + t_L)^6}{36} - \\
\frac{(t_\alpha + t_L)^2}{6}\left(3(t_1 + t_L) - 2(t_\alpha + t_L)\right) - \\
\frac{\alpha_1 (t_\alpha + t_L)^2}{60}\left(5(t_1 + t_L)^3 - 2(t_\alpha + t_L)^3\right) - \\
\frac{\alpha_4 (t_\alpha + t_L)^3}{12}\left(4(t_1 + t_L) - 3(t_\alpha + t_L)\right) - \\
\frac{\alpha_4\alpha_1 (t_\alpha + t_L)^3}{36}\left(2(t_1 + t_L)^3 - (t_\alpha + t_L)^3\right) - \\
\frac{\alpha_1 (t_\alpha + t_L)^4}{40}\left(5(t_1 + t_L) - 4(t_\alpha + t_L)\right)
\end{array} \right] \right\}
\end{array} \right\}
$$

Appendix B

$$
K(t_1+t_L)=\frac{1}{(T+t_L)}\left[\begin{array}{l}
\alpha_{oc}+\alpha_{hcow}\left[\begin{array}{l}
\alpha_w\left((t_\alpha+t_L)-\frac{\alpha_4(t_\alpha+t_L)^2}{2}-\frac{\alpha_1(t_\alpha+t_L)^3}{6}\right)\\
+\alpha_o e^{-\alpha_2((t_\alpha+t_L)+\alpha_4)}\left\{\begin{array}{l}
\frac{(t_1+t_L)^2}{2}-\frac{\alpha_4(t_1+t_L)^3}{6}+\frac{\alpha_1(t_1+t_L)^4}{12}\\
-\frac{\alpha_4\alpha_1(t_1+t_L)^5}{20}-\frac{(t_\alpha+t_L)^2}{2}\left(2(t_1+t_L)-(t_\alpha+t_L)\right)\\
-\frac{\alpha_1(t_\alpha+t_L)}{24}\left(4(t_1+t_L)^3-(t_\alpha+t_L)^3\right)\\
+\frac{\alpha_4(t_\alpha+t_L)^2}{6}\left(3(t_1+t_L)-2(t_\alpha+t_L)\right)\\
+\frac{\alpha_4\alpha_1(t_\alpha+t_L)^2}{30}\left(5(t_1+t_L)^3-3(t_\alpha+t_L)^3\right)\\
+\frac{\alpha\alpha_1(t_\alpha+t_L)^3}{24}\left(4(t_1+t_L)-3(t_\alpha+t_L)\right)
\end{array}\right\}
\end{array}\right]\\
+\alpha_{hcrw}\alpha_o\left[\begin{array}{l}
\frac{(t_\alpha+t_L)^2}{2}-\frac{(3\alpha_2+\alpha_4)}{6}(t_\alpha+t_L)^3\\
+\left(\frac{\alpha_1}{12}+\frac{\alpha_2\alpha_4}{8}\right)(t_\alpha+t_L)^4-\left(\frac{\alpha_4\alpha_1}{20}-\frac{\alpha_2\alpha_1}{30}\right)(t_\alpha+t_L)^5
\end{array}\right]\\
+\alpha_{dc}\alpha_1\left[\begin{array}{l}
\left(\alpha_o\left(\begin{array}{l}
\frac{1}{6}(t_\alpha+t_L)^3-\left(\frac{\alpha_2}{4}+\frac{\alpha_4}{12}\right)(t_\alpha+t_L)^4\\
+\left(\frac{\alpha_1}{40}+\frac{\alpha_4\alpha_2}{15}\right)(t_\alpha+t_L)^5-\left(\frac{\alpha_4\alpha_1}{36}-\frac{\alpha_2\alpha_1}{24}\right)(t_\alpha+t_L)^6
\end{array}\right)\right)\\
+\alpha_w\left(\frac{(t_\alpha+t_L)^2}{2}-\frac{\alpha_4(t_\alpha+t_L)^3}{3}\frac{\alpha_1(t_\alpha+t_L)^4}{8}\right)+\\
\alpha_o e^{-(t_\alpha+t_L)(\alpha_2+\alpha_4)}\left(\begin{array}{l}
\frac{(t_1+t_L)^3}{6}-\frac{\alpha_4(t_1+t_L)^4}{12}+\frac{\alpha_1(t_1+t_L)^5}{40}\\
-\frac{\alpha_4\alpha_1(t_1+t_L)^6}{36}-\frac{(t_\alpha+t_L)^2}{6}\left(3(t_1+t_L)-2(t_\alpha+t_L)\right)\\
-\frac{\alpha_1(t_\alpha+t_L)^2}{60}\left(5(t_1+t_L)^3-2(t_\alpha+t_L)^3\right)\\
-\frac{\alpha_4(t_\alpha+t_L)^3}{12}\left(4(t_1+t_L)-3(t_\alpha+t_L)\right)\\
-\frac{\alpha_4\alpha_1(t_\alpha+t_L)^3}{36}\left(2(t_1+t_L)^3-(t_\alpha+t_L)^3\right)\\
-\frac{\alpha_1(t_\alpha+t_L)^4}{40}\left(5(t_1+t_L)-4(t_\alpha+t_L)\right)
\end{array}\right)
\end{array}\right]
\end{array}\right]
$$