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# INTRODUCTION TO SOME CONTRIBUTION OF PARAMETER ESTIMATION, RELIABILITY AND QUALITY CONTROL IN STATISTICS

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**Abstract:** While hardware, software, and procedures are all considered when discussing a device's reliability, software program reliability is a crucial consideration because we're always on the lookout for high-quality and efficient software. A key component of software is dependability, which is useful for forecasting the exact software's level of creditability for the given period of time and under specific conditions. The reliability models can be divided into two types: dynamic models that track the fleeting behavior of debugging techniques during the checking out section, and static models that perform software logic modeling and evaluation at the same code. However, the nonlinear relationships between the model's parameters make it difficult to find the optimal parameters using conventional methods like maximum likelihood and least rectangular estimation and also the quality control. Numerous algorithms have been developed, which simplifies the process of estimating parameters. Numerous algorithms have been developed, which simplifies the process of estimating parameters

***Keywords:*** *Estimation of parameters, Reliability, Quality Control.*

## Introduction

Mechanical/Biological systems during the time of operation are always subjected to continuous environment stresses and shocks. Because of these strains, it is observed that the hazard rate behaviour of a system over its complete lifetime may not be uniform. It may follow some specific behavior up to a certain time point and changes thereafter. The time at which the change occurs is seldom known but it is of great importance as it can indicate the possibility of transition in the operative state of the system. This problem, to some extent, has been attempted through mixed and composite lifetime models by Bain (1978), Lawless (2003), and Sinha (1986). However, an assumption of mixed and composite lifetime distributions for figuring the change time point for hazard-rate seems unrealistic given the following facts-

- There are numerous physical causes that individually or collectively lead to the failure of a system at a time epoch.
- Practically, it is difficult to differentiate among these failure causes and to express their effect mathematically. Hence, choosing a suitable lifetime distribution for modeling the irregularities in the hazard-rate is an arduous task. In order to overcome this tricky situation, the studies like Mann, Schaffer and Singpurwalla (1974), Bain (1978), Sharma, Krishna and Singh (1997) have advocated the concept of hazard-rate that describes the instantaneous risk of the system's failure at time t given that the system was operative up to that time.

Thus is the Statistical inference about parameters of probability models has two basic features.

1. Estimation (point and interval)

2. Testing of hypotheses.

These procedures are heavily dependent upon the nature of the sample or design of the experiment. An assumption made in statistical inference is that a fixed random sample is available to the decision-maker. There are, however, many situations in practice where such as assumption cannot be made.

**Reliability**

In reliability studies it is natural to study a system with several components and wait until a failure occurs. Sometimes early failure data may not be available, or it may be impossible to wait until the whole system fails; rather the experiment is terminated after some fixed maximum amount of time. Such samples are called censored. Under various conditions of censoring, the estimation of parameters becomes involved even when one is using classical methods of estimation.

Two of the most common methods of estimation of parameters are maximum likelihood and least squares. The method of maximum likelihood requires maximization of the likelihood function such as product of probability density functions evaluated at each sample observation, while the least-squares method requires minimization of the sum of squared differences of the observed frequencies from those expected under the model. Optimization techniques, therefore, naturally enter both estimation processes.

A numerical optimizing technique plays an important role in cases where the solutions of the problems cannot be obtained in closed form. In cases of censored data and those of complicated models, optimization has to be done numerically by means of some iterative procedure. Constrained optimization results in models where parameters must satisfy certain conditions. For example, non-negativity restrictions on the parameters of most distributions, or probabilities summing to one for each row of a Markov transition matrix. Another example here refers to the use of mixtures of distributions as a model for a given phenomenon, in which case the mixture condition is that the probabilities with which individual probability distributions enter into the model add to one. Many of the above optimization problems reduce to those of mathematical programming. A collection of mathematical programming techniques applied to statistical problems can be found in Arthanari and Dodge [1]. Various numerical methods and algorithms have been provided by Kennedy and Gentle [5] and Bard [2] for statistical problems.

In the paper "Parameter estimation under progressive censoring conditions for a finite mixture of Weibull distributions", Mendelbaum and Harris give algorithms for finding maximum likelihood estimates under different sampling environments. The model chosen is that of a mixture of several Weibull distributions under progressive censoring, most often encountered in reliability applications. Maximum likelihood estimates have desirable asymptotic properties, but such estimators are not available in closed form for several important distributions, including the Weibull. Iterative procedures are therefore needed, which often experience computational difficulties when all three Weibull parameters are unknown (Zanakis, [10, 11]). Progressive censoring is concerned with observing objects for an arbitrary interval of time and noting the time of their failure. Therefore two measurements, the arbitrary interval for observation time and time to failure, are obtained for each individual object. The likelihood function in such a case is nonlinear, subject to linear constraints, and thus results in a mathematical programming problem. The applications are made not only to problems in avionic equipment (Mendenhall and Hader [7]) but also to problems in criminal justice program evaluation (Carr-Hill and Carr-Hill [3]). The problems arising from tests of hypotheses of such parameters are still open in the above mixed models.

Data often include observations which are suspicious. The maverick observations may belong naturally to the sample and reflect the distribution as such; or the observations may be genuinely bad ones and should be removed from the data before making any inference. Such observations have been called outliers and there have been several tests in statistics to study outliers, including some developed especially for time-series forecasting and process quality control. Mann, in her paper "Optimal outlier tests for a Weibull model to identify process changes or to predict failure times", provides a comparison of three different statistics for testing outliers. The properties of the tests are studied and it is shown that one of the statistics proposed gives the most powerful test under certain situations.

Most of the classical tests for outliers have been developed for the normal model. In reliability and some areas of medicine, where Weibull distributions are commonly used, there is a scarcity of such tests. It should be noted that there are distributions which are outlier-prone, even though the variance is finite. Several such distributions have been given by Neyman and Scott [8], where they were concerned with studying weather data in cloud-seeding experiments. They have brought out the distinctions between cases where the tendency to suspect and to eliminate outlier observations may be justifiable and those in which it is not. They show that the log normal and gamma distributions are outlier-prone.

In reliability theory, applications occur frequently where the probability distributions have a decreasing failure rate. Failure (hazard) rate is commonly defined in terms of probability distribution of the time to failure. If $f(x)$ is the probability density function of the time to failure and $F(x)$; its cumulative distribution function, then

$$h(x) = \frac{f(x)}{[1-F(x)]}$$, defines the failure rate.

Then $h(x)\,dx$ is the conditional probability of failure in the interval $(x,\ x+dx)$ given that the individual has survived till time $x$. For a Weibull distribution with shape parameter $c$, the failure rate function $h(x)$ is decreasing rapidly for $c<1$, constant for $c=1$, and increasing for $c>1$, with an increasing $(c>2)$ constant $(c=2)$ or decreasing $(1<c<2)$ slope. This flexibility of the

Weibull model is probably the primary reason for its wide use in practice. For an extensive discussion of reliability theory, the reader is referred to Mann, Schaefer and Singpurwalla [6].

In their paper "Extreme points of the class of discrete decreasing failure rate average life distributions", Langberg, Leon, Lynch and Proschan show that the classes of distributions which are discrete with decreasing failure rate average form a convex set. Such distributions can be represented in terms of the extreme points of the convex set described earlier. Several other aspects of these distributions can also be discussed in terms of the extreme points of the convex set.

Singpurwalla studies the estimation of reliability growth in his paper "A Bayesian scheme for estimating reliability growth under exponential failure times". When the reliability of a system is studied at several stages, the procedure is called reliability growth. Using an inverted gamma distribution as the prior distribution in the parameter of the exponential distribution at each stage, the estimate for the mean time to failure is obtained in terms of the posterior distribution. The procedure is especially relevant when the posterior distributions are stochastically ordered, but one situation is also discussed when this is not the case.

**Quality Control:-**

Acceptance sampling forms an important area of quality control methodology. In single sampling plans one must determine the sample size and acceptance number (number of defective items in the sample) to achieve a certain level of confidence. In their paper "A bicriterion model for acceptance sampling" Moskowitz, Ravindran, Klein and Eswaran use a criterion with two 0bJectives to solve the above problem; namely to maximize a measure of outgoing lot quality and minimize costs. For a detailed treatment of multiobJective optimization theory and application the reader is referred to Hwang and Masud [4]. See also Starr and Zeleny [9] for a good exposition on multi-attribute utility and preference. An implicit enumeration algorithm and an interactive paired-comparisons nonlinear optimization procedure are given by Moskowitz et al. for the sampling problems considered. Simulation experiments are performed to determine the robustness of the sampling plan and the response surfaces of the bicriterion model to various utility functions.

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