Journal of Research Administration
Society of Research Administrators International

# AN EOQ MODEL FOR INSTANTANEOUS DETERIORATING ITEMS WITH HYBRID PRICE & STOCK DEPENDENT DEMAND WITH PARTIAL BACKLOGGING & ADVANCE PAYMENT RELATED DISCOUNT FACILITIES UNDER TWO LEVEL TRADE CREDIT PERIOD

**Dr.D.Chitra[1], R.Sandhya[2]**

[1] Associate Professor, PG & Research Department of Mathematics, Quaid-E-Millath Government College for Women (Autonomous), Anna salai, Chennai-02, Tamilnadu, India.

[2]Research Scholar, PG & Research Department of Mathematics, Quaid-E-Millath Government College for Women (Autonomous), Anna salai, Chennai-02, Tamilnadu, India

## ABSTRACT

In this paper an EOQ model for instantaneous deteriorating items with hybrid price and stock dependent demand on selling price and stock is developed. The concept of pre-payment policy with a discount facility and two-level of trade credit policy is introduced. The retailer who purchases the items enjoys a fixed credit period offered by the supplier and in turn, also offers a credit period to the customer in order to attract the customers. Shortages are allowed and partially backlogged. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. The classical optimization technique is used to obtain the average profit of the proposed model as a non-linear maximization problem. The main objective of the inventory model is to maximize the total profit per unit time of the retailer by offering discount facility on the purchase price in the case of pre-payment by the customers and offering credit period for both retailer and customer in order to promote the market competition. Finally numerical examples and sensitivity analysis of the major parameters are presented to illustrate the developed model.

**Keywords:** Inventory, instantaneous deterioration, partial backlogging, hybrid price and stock demand, discount facilities, advance prepayment, Two-Level trade credit period.

## 1. INTRODUCTION

Inventory control is essential for companies to reduce their costs, maintaining stock, improving products quality, providing better services and managing customer demands, companies are facing greater challenges when they are working with deteriorating products. Large quantity of goods displayed in market lure the customers to buy more. If the stock is insufficient, customer may prefer some other brand, as a result it will fetch loss to the supplier. In some inventory systems such as fashionable commodities the length of waiting time for the next replenishment is the main factor in determining whether backlogging will be accepted or not, the longer the waiting time is the smaller the backlogging rate would be and vice versa.

In the theory of inventory management, customers demand is a key parameter of any inventory model. The nature of demand depends on many factors viz., selling price, availability of the stock, time, quality of the product, ecofriendliness of the product, impreciseness, etc. Thus, the researchers or decision makers take the demand rate as a hybrid form of price or the inventory level or both hybrids selling price and stock or time or green level, etc. Sana and Chaudhuri [1] studied the optimal policy of an inventory model based on the assumptions of the demand as both the stock level and advertisement dependent whereas Balkhi and Tadj [2] analyzed a model for perishable items along with the demand of the product varying with time.. Later, Pal et al. [3] studied an inventory model with ramp type demand of the product. Khan et al. [4] delineated the optimal inventory policy under linear price decreasing demand function and all-units discount. Recently, Shaikh et al. [5] investigated another model for decaying goods with inventory level-related market demand under partial backlogging in a price discount environment. Considering the uncertainty into account, Rahman et al., analyzed an imprecise model for decaying goods where the demand for the product is associated with the interval valued market price.

Pre-payment or advanced payment is another most popular business policy in the area of business management. Sometimes it happens that due to the higher demand for a product and it's an inadequate supply, the stock level of that particular product is volatized in the market. To manage such cases, the buyers desire to prepay either the full purchasing cost or a fixed percentage of full purchasing cost to make sure the guarantee of the on-time delivery of that particular commodities. The idea of advanced payment was first introduced model by Zhang [6] in the yard of business management. After that lot of research works were accomplished by eminent researchers. Among those, some worth-mentioning works are presented here. Maiti et al. [7] analyzed the positive effect of the prepayment on the inventory system by using a genetic algorithm (GA) optimizers and generalized reduced gradient (GRG) technique. Thangam [8] studied optimal lot-sizing policy for the deteriorating items under advance payment strategy. Zhang et al. [9] developed an EOQ model under a pre-payment and delay payment scheme simultaneously. Similarly, Zia and Taleizadeh [10] studied another lot-sizing model with a delayed and advanced payment scheme. Li et al. [11] found the pricing and lot-sizing policies for deteriorating goods under an advance payment scheme. Recently, Khan et al. [12], Shaikh et al.[13], Shi et al. [14], Qin et al., Khan et al., Md S Rahman [15] and others worked on the advanced payment policy.

Permissible delay in payment is one of the most important and popular strategies in the business world. With this strategy, the suppliers attract more customers, and therefore more sales of the product. The retailers then accept this and also give the facility of a delay in payment to their customers. All this is done with the aim of attracting more customers, and consequently selling more products. The credit policy consists of giving to a buyer the payment facility of paying the purchased amount up to a certain period. On the one hand, no interest is charged by the supplier to the retailer if the payment is done up to the credit period. On the other hand, an interest is charged by the supplier to the retailer when the credit period is over. Recently, several researchers have performed several kinds of research by including a twolevel trade credit concept. Goyal first formulated an economic order quantity (EOQ) model by incorporating the condition of permissible delay in payments. Subsequently, many researchers have studied the inventory models under the condition of delay in payments. Several interesting articles developed under the condition of delay in payments are Teng et al. [16], Sarkar [17] and Min et al. [18] and the references therein. Later, Huang [19] extended Goyal's model and established optimal retailer's ordering policies in the

EOQ model when the supplier offers trade credit to the retailer and the retailer will also adopt trade credit policy for his/her customers. This phenomenon is known as two-level trade credit. Thereafter, Huang extended the inventory model with the trade credit for a supply chain system with both an upstream and a downstream credit. Teng and Goyal [20] modified the assumption of Huang's inventory model by introducing the concept that the retailer obtains its revenue from N to N + T, not from 0 to T. They also stated that the unit selling price is higher than the unit purchased cost. At the same time, Teng [21] obtained the optimal ordering policy for the retailer to deal with credit risk clients, as well as good credit clients. Min et al. worked on an inventory model under stock dependent demand and two-level trade credit. In this line of research, there exist the following papers in the area of Sustainability 2021, 13, 13493 3 of 19 the two-level trade credit policy such as Kreng and Tan [22], Teng and Lou [23] Mahata [24], Chung and Cárdenas-Barrón [25], Chung et al. [26], Chen et al. [27], Shah and Cárdenas-Barrón [28] and others. For example, Jaggi et al. Aliabadi et al. [29] solved a non-instantaneous deteriorating inventory problem with credit period and carbon emissions dependent demand by using a geometric programming approach. Liao et al. [30] addressed an EOQ inventory model with a delay in payment policy for non-instantaneous deteriorating items with the aim of finding an optimal ordering policy. Shaikh et al. [31] considered price-sensitive demand, inflation, and reliability. Recently Abu Hashan Md Mashud [32] worked on integrated price-sensitive inventory model for deteriorating items under trade credit policy.

In this work, we have studied an EOQ model for instantaneous deteriorating items under the consideration of hybrid market price and stock level dependent demand along with prepayment and discount facility under two level trade credit period. The shortages is allowed and partially backlogged with a constant backlogging rate. Here, the discount facility is measured on the purchase price and on the basis of pre-payment to entice more customers under the prepayment scheme as follows: if a practitioner pays the whole purchase price before getting the delivery at the order placing moment, on the basis of prepayment a certain percentage discount is offered by the supplier on the total purchase price. In the two-level trade credit system, the supplier offers a trade credit facility to the retailer and then retailer also provides a trade credit to the customers. The proposed model is formulated mathematically by using ordinary differential equations and the corresponding optimization is obtained as a profit maximization problem.

## 2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been used in the entire paper.

### 2.1 ASSUMPTIONS:

1. The inventory system involves one item.
2. The replenishment rate is infinite and lead time is zero.
3. There is no replacement or repair of deteriorated units.
4. Hybrid price and stock dependant demand is assumed to develop this proposed model

and its mathematical form can be expressed as

$$D\{I(t),p\}=\begin{cases}\alpha(p)+\beta I(t), when I(t)>0\\ \alpha(p), when I(t)\le 0\end{cases}$$

Where $\alpha(p)=\varphi(a-bp)+(1-\varphi)cp^{-\lambda}, \beta>0, a-bp\ge 0, 0\le\varphi\le 1, 0<\lambda<1.$

5. Shortages are allowed and the backlogged rate is defined to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter $\delta$ is a positive constant.
6. The fixed credit period offered by the supplier to the retailer is no less than the credit period permitted by the retailer to the customer (ie) $M\ge N$
7. When $T\le M$ , the account is settled at $t=M$ and the retailer does not need to pay any interest charge of items in stock during the whole cycle.
8. The retailer can accumulate revenue and earn interest during the period from $t=N$ to $t=M$ with rate $I_e$ under the condition of trade credit.

**2.2 NOTATIONS:**

1. $I(t)$ - Inventory level at any time t where $0\le t\le T$
2. $D(t)$ - Demand rate function.
3. $\theta$ - Deterioration rate, $0<\theta<1$.
4. $P$ **-** Purchase cost, $ per unit
5. $K$ **-** The Replenishment cost per order ($/order)
6. $h$ - Holding cost per unit per unit time
7. $s$ - Shortage cost, $ per unit per unit time
8. $p$ - Selling price, $ per unit per unit time
9. $\pi$ - Opportunity cost due to lost sale, $ per time per unit time
10. $\delta$ - The Backlogging parameter (a positive constant) $0<\delta<1$ .
11. $d$ - Discount percentage based on the advance payment %
12. $Q$ - Lot size of total order
13. $t_1$ - Time at which shortages starts, $0\le t_1\le T$ .
14. $T$ - The length of the Replenishment cycle
15. S - Maximum product amount at the very beginning of the cycle
16. R - Maximum storage amount at the end of the cycle
17. $M$ - The retailer's trade-credit period offered by the supplier
18. $N$ - The customer's trade-credit period offered by the retailer
19. $I_e$ - Interest which can be earned per $ per year by the retailer
20. $I_r$ **-** Interest charges per $ in stocks per year by the supplier
21. $TP(t_1,T)$ - The total profit, $ per unit time

## 3. MATHEMATICAL FORMULATION:

Based on the assumptions mentioned earlier, this section presents the following inventory model formulation. In the beginning, an enterprise purchase Q units of products by paying the entire purchase price at a time prior to L unit time from the receiving moment and then in return to prepayment he enjoys a certain discount on the entire price. At time $t = t_1$, the inventory level reaches zero and shortages occurs. During the shortage interval $(t_1, T)$, the demand rate at time t is partially backlogged. During the stock-out period, the backlogged rate is variable and is dependent on the length of the waiting time for the next replenishment. So the backlogged rate for negative inventory is donated as $\frac{1}{1+\delta(T-t)}$, where $\delta$ is the backlogging parameter $0 < \delta < 1$.

Hence the inventory level $I(t)$ is given by

$$\frac{dI_1(t)}{dt} = -\alpha(p) - \beta I(t) - \theta I(t), 0 \le t \le t_1$$

$$\frac{dI_2(t)}{dt} = -\frac{\alpha(p)}{1+\delta(T-t)}, t_1 \le t \le T \qquad (1)$$

With boundary conditions $I(t_1) = 0$ and $I(t)$ is continuous at $t = t_1$.

The solutions of the above differential equation are

$$I_1(t) = -\frac{\alpha(p)}{\beta+\theta} + \frac{e^{-(\beta+\theta)t}\alpha(p)}{e^{-(\beta+\theta)t_1}(\beta+\theta)} \qquad (2)$$

$$I_2(t) = \frac{\alpha(p)\log(1+\delta(T-t)) - \alpha(p)\log(1+\delta T - \delta t_1)}{\delta} \qquad (3)$$

Exploiting the boundary condition $I_1(0) = S$, we get

$$S = -\frac{\alpha(p)}{\beta+\theta} + \frac{\alpha(p)}{e^{-(\beta+\theta)t_1}(\beta+\theta)} \qquad (4)$$

Again exploiting the boundary condition $I_2(T) = -R$, we get

$$R = \frac{\alpha(p)\log(1+\delta T - \delta t_1)}{\delta} \qquad (5)$$

The total number of ordered amount is

$$Q = S + R \qquad (6)$$

The total inventory profit consists of the following components.

a) Ordering cost per cycle is K. (7)

b) Inventory holding cost HC per cycle is given by

c) $HC = h\int_0^{t_1} I_1(t)dt$

$$= \frac{h\alpha(p)\left(e^{t_1\beta+t_1\theta} - 1 - t_1\beta - t_1\theta\right)}{\beta^2 + 2\beta\theta + \theta^2} \qquad (8)$$

d) The shortage cost in the interval $[t_1, T)$ denoted by SC is given by,

$$SC = s\int_{t_1}^{T} -I_2(t)dt$$

$$= -\frac{s\alpha(p)\left(\log(1+\delta T-\delta t_1)-\delta T+\delta t_1\right)}{\delta^2} \tag{9}$$

e) The opportunity cost due to lost sales denoted by OC is given by,

$$OC = \pi\int_{t_1}^{T}\alpha(p)\left[1-\frac{1}{1+\delta(T-t)}\right]dt$$

$$= \pi\alpha(p)T - \pi\left[\alpha(p)t_1 + \frac{\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right] \tag{10}$$

f) The Purchasing cost for the first replenishment cycle by PC is given by

$$PC = (1-d)P(S+R)$$

$$= (1-d)P\left[-\frac{\alpha(p)}{\beta+\theta}+\frac{\alpha(p)}{e^{-(\beta+\theta)t_1}(\beta+\theta)}+\frac{\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right] \tag{11}$$

a) The capital cost denoted by CC is

$$CC = LI_e PC$$

$$= LI_e(1-d)P\left[-\frac{\alpha(p)}{\beta+\theta}+\frac{\alpha(p)}{e^{-(\beta+\theta)t_1}(\beta+\theta)}+\frac{\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right] \tag{12}$$

b) Sales revenue denoted by SR is

$$SR = p\int_{0}^{t_1}\left[\alpha(p)+\beta I_2(t)\right]dt + pR$$

$$= \left[\frac{p\alpha(p)\left(\beta e^{t_1\beta+t_1\theta}-\beta+t_1\beta\theta+t_1\theta^2\right)}{\beta^2+2\beta\theta+\theta^2}+\frac{p\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right] \tag{13}$$

Therefore the total inventory profit $(X)$= Sales revenue – Ordering cost – Holding cost – Shortage cost – Opportunity cost – Purchasing cost – Capital cost

i.e. $X = SR - K - HC - SC - OC - PC - CC$

$$
=\left\{\begin{array}{l}
\frac{p\alpha(p)\left(\beta e^{t_1\beta+t_1\theta}-\beta+t_1\beta\theta+t_1\theta^2\right)}{\beta^2+2\beta\theta+\theta^2}+\frac{p\alpha(p)\log(1+\delta T-\delta t_1)}{\delta} \\
-K-\frac{h\alpha(p)\left(e^{t_1\beta+t_1\theta}-1-t_1\beta-t_1\theta\right)}{\beta^2+2\beta\theta+\theta^2}+\frac{s\alpha(p)\left(\log(1+\delta T-\delta t_1)-\delta T+\delta t_1\right)}{\delta^2} \\
-\pi\alpha(p)T-\pi\left[\alpha(p)t_1+\frac{\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right] \\
+(1-d)P\left[-\frac{\alpha(p)}{\beta+\theta}+\frac{\alpha(p)}{e^{-(\beta+\theta)t_1}(\beta+\theta)}+\frac{\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right] \\
LI_e(1-d)P\left[-\frac{\alpha(p)}{\beta+\theta}+\frac{\alpha(p)}{e^{-(\beta+\theta)t_1}(\beta+\theta)}+\frac{\alpha(p)\log(1+\delta T-\delta t_1)}{\delta}\right]
\end{array}\right\} \quad (14)
$$

Considering the permissible delay period M for supplier offered by the Retailer and N for customer offered by the supplier, the inventory model has the following cases

**Case 1:** $M \le t_1$

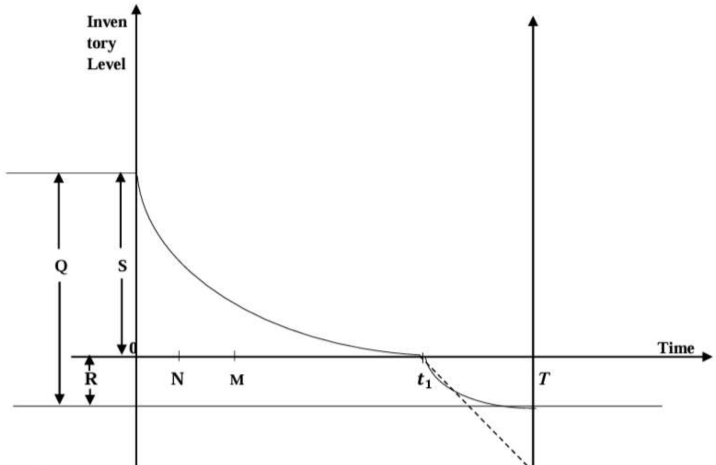


Figure 1:

This case can be extended as $0 \le N \le M \le t_1$, in which both N and M is shorter than $t_1$. The interest payable in this case is

$$
IP_{11}=pI_r\left(\int_M^{t_1} I_1(t)dt\right)
$$

$$
=\left\{\begin{array}{l}
\frac{1}{\beta^2+2\beta\theta+\theta^2}\left[PI_r\left(-\beta e^{-(\beta+\theta)t_1}t_1+\beta Me^{-(\beta+\theta)t_1}-e^{-(\beta+\theta)t_1}+Me^{-(\beta+\theta)t_1}\theta-e^{-(\beta+\theta)t_1}t_1\theta\right.\right. \\
\left.\left.+e^{-(\beta+\theta)M}\right)\alpha(p)e^{-(\beta+\theta)t_1}\right]
\end{array}\right\} \quad (15)
$$

The interest earned in this case is

$$IE_{11} = pI_e\left(\int_N^M I_1(t)tdt\right)$$

$$= \begin{cases} -\frac{1}{2}\frac{1}{(\beta+\theta)^3}\left[pI_e\alpha(p)\left(-N^2\beta^2 - 2N^2\beta\theta - N^2\theta^2 - 2e^{t_1\beta+t_1\theta-N\beta-N\theta}N\beta - 2e^{t_1\beta+t_1\theta-N\beta-N\theta}N\theta\right.\right. \\ -2e^{t_1\beta+t_1\theta-N\beta-N\theta} + M^2\beta^2 + 2M^2\beta\theta + M^2\theta^2 + 2e^{t_1\beta+t_1\theta-\beta M-M\theta}\beta M \\ \left.\left.+2e^{t_1\beta+t_1\theta-\beta M-M\theta}M\theta + 2e^{t_1\beta+t_1\theta-\beta M-M\theta}\right)\right] \end{cases} \quad (16)$$

**Case 2:** $M \geq t_1$

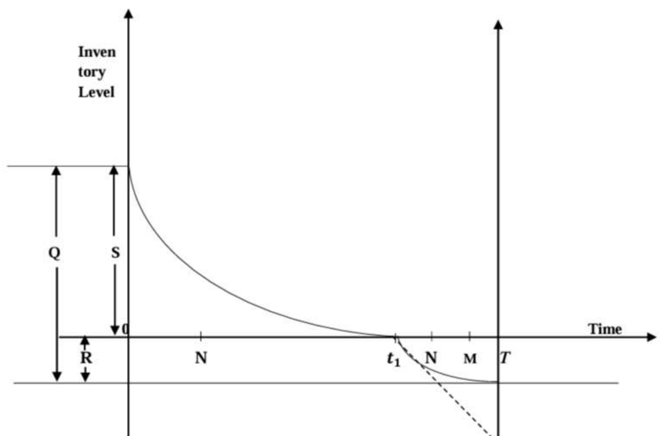


Figure 2:

In this case, two sub-cases may arise, $0 \leq N \leq t_1 \leq M$ and $0 \leq t_1 \leq N \leq M$

**Subcase 21:** $0 \leq N \leq t_1 \leq M$

The interest payable in this case is 0

$$IP_{21} = 0 \quad (17)$$

The interest earned in this case is

$$IE_{21} = pI_e\left(\int_N^{t_1} I_1(t)tdt + \int_{t_1}^M I_2(t)tdt\right)$$

$$= \begin{cases} pI_e\left\{\frac{1}{4}\frac{1}{\delta^3}\left[\alpha(p)\left(-2\log(1+\delta T-\delta t_1) - 4\log(1+\delta T-\delta t_1)\delta T + 4\log(1+\delta T-\delta N)T\delta\right.\right.\right. \\ +2\log(1+\delta T-\delta N)T^2\delta^2 - 2\log(1+\delta T-\delta N)N^2\delta^2 + 2TN\delta^2 - 2\log(1+\delta T-\delta t_1)\delta^2T^2 \\ \left.\left.+2\log(1+\delta T-\delta t_1)N^2\delta^2 + 2\delta N + N^2\delta^2 + 2\log(1+\delta T-\delta N) - 2\delta^2Tt_1 - 2\delta t_1 - \delta^2t_1^2\right)\right] \\ -\frac{1}{4}\frac{1}{\delta^3}\left[\alpha(p)\left(-2\delta^2Tt_1 - 2\delta t_1 - \delta^2t_1^2 - 2\log(1+\delta T-\delta t_1) - 4\log(1+\delta T-\delta t_1)\delta T\right.\right. \\ +4\log(1+\delta T-\delta M)T\delta + 2\log(1+\delta T-\delta M)T^2\delta^2 - 2\log(1+\delta T-\delta M)M^2\delta^2 + 2TM\delta^2 \\ \left.\left.\left.-2\log(1+\delta T-\delta t_1)\delta^2T^2 + 2\log(1+\delta T-\delta t_1)M^2\delta^2 + 2\delta M + M^2\delta^2 + 2\log(1+\delta T-\delta M)\right)\right]\right\} \end{cases} \quad (18)$$

**Subcase 22:** $0 \le t_1 \le N \le M$

The interest payable in this case is 0

$IP_{22} = 0$

The interest earned in this case is

$$IE_{22} = pI_e\left(\int_N^M I_2(t)tdt\right)$$

$$= \left\{ \begin{array}{l} -\frac{1}{4}\frac{1}{\delta^3}\left[pI_e\alpha(p)\left(-4\log(1+\delta T-\delta N)T\delta - 2\log(1+\delta T-\delta N)T^2\delta^2 + 2\log(1+\delta T-\delta N)N^2\delta^2 \right.\right. \\ -2TN\delta^2 - 2\log(1+\delta T-\delta t_1)N^2\delta^2 + 4\log(1+\delta T-\delta M)T\delta + 2\log(1+\delta T-\delta M)T^2\delta^2 \\ -2\log(1+\delta T-\delta M)M^2\delta^2 + 2TM\delta^2 + 2og(1+\delta T-\delta t_1)M^2\delta^2 - 2\delta N \\ \left.\left. -N^2\delta^2 - 2og(1+\delta T-\delta N) + 2\delta M + M^2\delta^2 + 2og(1+\delta T-\delta M)\right)\right] \end{array} \right\} \quad (19)$$

**Total Profit Function:**

$$TP_i(T,t_1) = \begin{cases} TP_1, 0 \le N \le M \le t_1 \\ TP_2, 0 \le N \le t_1 \le M \\ TP_3, 0 \le t_1 \le N \le M \end{cases} \quad (20)$$

$$TP_1(T,t_1) = \frac{1}{T}\left[X + IE_{11} - IP_{11}\right] \quad (21)$$

$$TP_2(T,t_1) = \frac{1}{T}\left[X + IE_{21} - IP_{21}\right] \quad (22)$$

$$TP_3(T,t_1) = \frac{1}{T}\left[X + IE_{22} - IP_{22}\right] \quad (23)$$

The necessary conditions for the total profit $\partial TP_i(T,t_1)$ is concave with respect to $T$ and $t_1$ are

$$\frac{\partial TP_i(T,t_1)}{\partial T} = 0 \text{ and } \frac{\partial TP_i(T,t_1)}{\partial t_1} = 0$$

Provided they satisfy the sufficient conditions $\left.\frac{\partial^2 TP_i(T,t_1)}{\partial T^2}\right|_{(T^*,t_1^*)} < 0, \left.\frac{\partial^2 TP_i(T,t_1)}{\partial t_1^{\,2}}\right|_{(T^*,t_1^*)} < 0$

and $\begin{bmatrix} \frac{\partial^2 TP_i}{\partial T^2} & \frac{\partial^2 TP_i}{\partial T \partial t_1} \\ \frac{\partial^2 TP_i}{\partial t_1 \partial T} & \frac{\partial^2 TP_i}{\partial t_1^{\,2}} \end{bmatrix} < 0$

We develop the following algorithm to find the optimal values of $T$ and $t_1$ (say $T^*, t_1^*$) that maximize $TP_i(T,t_1)$

## SOLUTION PROCEDURE:

The problem mentioned above is solved by using the following algorithm:

**Step1:** Start

**Step 2:** Plug all the value of the required parameters of the proposed model in the equation (20)

**Step 3:** Put $\frac{\partial TP_i}{\partial t_1} = \frac{\partial TP_i}{\partial T} = 0$, $i = 1,2,3$

**Step 4:** Solve the optimization problem $TP_i$ for $i = 1,2,3$ and store the optimal value of $t_1^*, T^*, TP^*, S^*$ and $R^*$

**Step 5:** Compare the value of $TP_1, TP_2$ and $TP_3$.

**Step 6:** Choose the maximum value among $TP_1, TP_2$ and $TP_3$.

**Step 7:** Stop

## NUMERICAL EXAMPLES:

**Example 1:** Consider the inventory system with the following data $K = 200, \varphi = 0.50, a = 50, b = 0.5, p = 25, c = 90, \lambda = 0.29, \theta = 0.08, \beta = 0.007, \delta = 0.22, P = 10, h = 4.6,$ $s = 12, \pi = 8, d = 0.3, L = 0.2, I_e = 0.11, I_r = 0.12, M = 1.876, N = 1.313$ in appropriate units. In this case $N \le M \le t_1$. Using the algorithm we obtain the optimal solution as $T = 2.8389$ months, $t_1 = 2.0464$ months. Hence the Total profit per unit time is $TP_1 = 157.3571$ **$,** $S = 40.7519$ quintals, $R = 13.2901$ quintals.

**Example 2:** Taking all the parameters same except $M = 2.436, N = 1.313$ in appropriate units. In this case $N \le t_1 \le M$. Using the algorithm we obtain the optimal solution as $T = 3.4512$ months, $t_1 = 2.1467$ months. Hence the Total profit per unit time is $TP_1 = 102.36424$ **$,** $S = 51.6812$ quintals, $R = 10.4539$ quintals

**Example 3:** Taking all the parameters same except $M = 2.7323, N = 2.2322$ in this case $t_1 \le N \le M$. Using the algorithm we obtain the optimal solution as $T = 3.6481$ months, $t_1 = 2.0436$ months. Hence the Total profit per unit time is $TP_1 = 117.8285$ **$,** $S = 41.0236$ quintals, $R = 12.6981$ quintals.

**Effect of change in various parameter of the inventory is presented in the following table**

| **Changing parameter** | **Change in parameter** | $t_1$ | $T$ | $TP$ | $S$ | $R$ |
|---|---|---|---|---|---|---|
| $K$ | 196 | 2.0074 | 2.7745 | 159.0579 | 39.9045 | 12.8970 |
| | 198 | 2.0175 | 2.7925 | 158.2380 | 40.1249 | 13.0182 |
| | 202 | 2.0373 | 2.8275 | 156.6050 | 40.5547 | 13.2550 |
| | 204 | 2.0470 | 2.8446 | 155.7920 | 40.7644 | 13.3706 |
| $p$ | 24.6 | 2.0147 | 2.7773 | 156.9303 | 40.2474 | 12.8845 |
| | 24.8 | 2.0277 | 2.7964 | 159.9900 | 40.4362 | 12.9510 |
| | 25.2 | 2.0536 | 2.8347 | 166.0579 | 40.8162 | 13.0851 |
| | 25.4 | 2.0666 | 2.8540 | 169.0663 | 41.0076 | 13.1526 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta$ | 0.076 | 2.0676 | 2.8728 | 157.6954 | 41.0387 | 13.4868 |
| | 0.078 | 2.0467 | 2.8399 | 157.5774 | 40.6717 | 13.3019 |
| | 0.082 | 2.0100 | 2.7832 | 157.2318 | 40.0438 | 12.9913 |
| | 0.084 | 1.9938 | 2.7587 | 157.0176 | 39.7748 | 12.8613 |
| $d$ | 0.296 | 2.0271 | 2.8106 | 156.6269 | 40.3337 | 13.1510 |
| | 0.298 | 2.0273 | 2.8104 | 157.0236 | 40.3377 | 13.1442 |
| | 0.302 | 2.0277 | 2.8099 | 157.8172 | 40.3456 | 13.1308 |
| | 0.304 | 2.0279 | 2.8096 | 158.2139 | 40.3495 | 13.1241 |
| $\varphi$ | 0.496 | 2.0350 | 2.8234 | 155.6135 | 40.1980 | 13.1271 |
| | 0.498 | 2.0313 | 2.8168 | 156.5168 | 40.2701 | 13.1325 |
| | 0.502 | 2.0238 | 2.8035 | 158.3243 | 40.4126 | 13.1423 |
| | 0.504 | 2.0200 | 2.7969 | 159.2285 | 40.4832 | 13.1468 |
| | 9.6 | 2.0406 | 2.8155 | 163.0325 | 40.6259 | 13.0179 |
| | 9.8 | 2.0340 | 2.8128 | 160.2262 | 40.4830 | 13.0778 |
| $P$ | 10.2 | 2.0211 | 2.8075 | 154.6150 | 40.2018 | 13.1971 |
| | 10.4 | 2.0147 | 2.8050 | 151.8100 | 40.6348 | 13.2566 |
| | 0.216 | 2.0287 | 2.8094 | 157.3304 | 40.3684 | 13.0893 |
| | 0.218 | 0.0281 | 2.8098 | 157.3753 | 40.3549 | 13.1133 |
| $\delta$ | 0.222 | 2.0269 | 2.8105 | 157.4655 | 40.3284 | 13.1619 |
| | 0.224 | 2.0263 | 2.8109 | 157.5108 | 40.3154 | 13.1865 |
| | 0.286 | 2.0512 | 2.7881 | 160.4351 | 40.5764 | 13.1525 |
| | 0.288 | 2.0213 | 2.7991 | 158.9225 | 40.4594 | 13.1453 |
| $\lambda$ | 0.292 | 2.0337 | 2.8211 | 155.9288 | 40.2232 | 13.1290 |
| | 0.294 | 2.0398 | 2.8320 | 154.4478 | 40.1043 | 13.1199 |
| | 1.872 | 2.0350 | 2.8216 | 157.3970 | 40.5048 | 13.1677 |
| | 1.874 | 2.0313 | 2.8158 | 157.4090 | 40.4231 | 13.1677 |
| M | 1.878 | 2.0238 | 2.8045 | 157.4312 | 40.2605 | 12.1076 |
| | 1.880 | 2.0200 | 2.7988 | 157.4413 | 40.1797 | 12.0778 |
| | 1.309 | 2.0366 | 2.8219 | 157.3890 | 40.5382 | 13.1801 |
| | 1.311 | 2.0415 | 2.8304 | 157.3739 | 40.6449 | 12.2351 |
| N | 1.315 | 2.0513 | 2.8474 | 157.3383 | 40.8592 | 13.3459 |
| | 1.317 | 2.0563 | 2.8559 | 157.3179 | 40.9667 | 13.4016 |

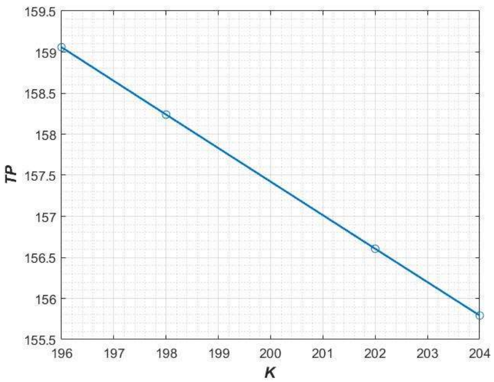


Fig.3. Effect of $K$ on Total Profit

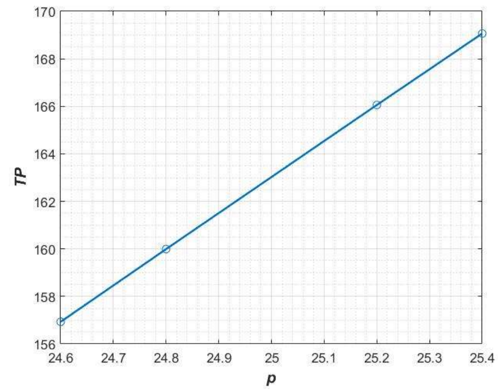


Fig.4. Effect of $p$ on Total Profit

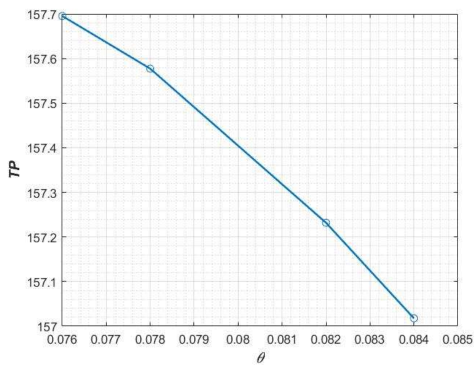


Fig.7. Effect of $\theta$ on Total Profit

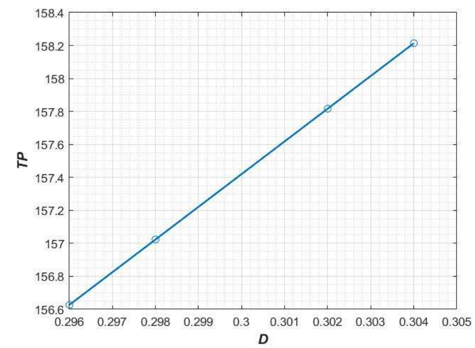


Fig.8. Effect of $d$ on Total Profit

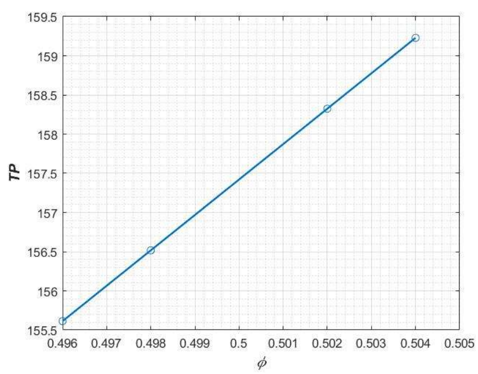


Fig.9. Effect of $\varphi$ on Total Profit

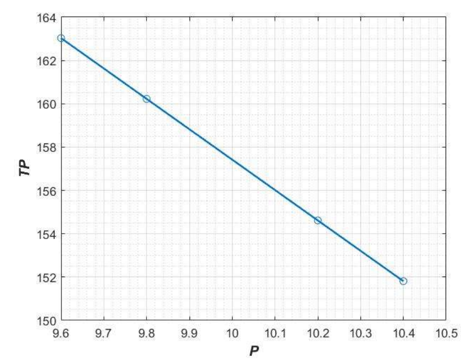


Fig.10. Effect of $P$ on Total Profit

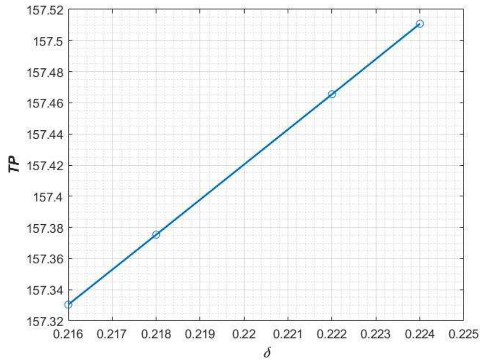


Fig.11. Effect of $\delta$ on Total Profit

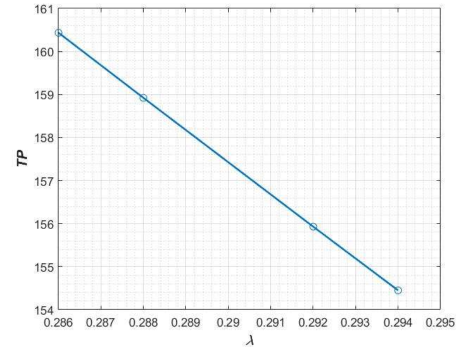


Fig.12. Effect of $\lambda$ on Total Profit

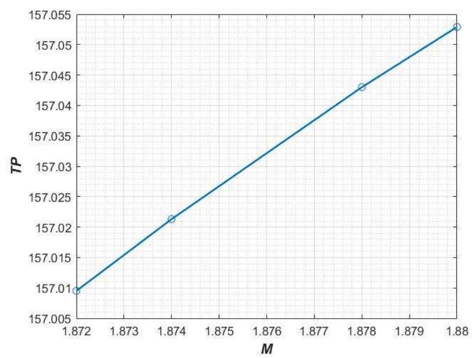


Fig.13. Effect of $M$ on Total Profit

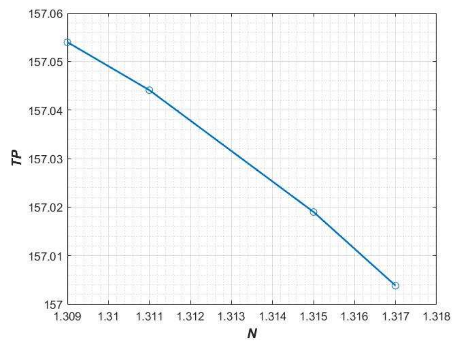


Fig.14. Effect of $N$ on Total Profit

## 4. SENSITIVITY ANALYSIS:

Now, let us study on changes in the values of the system parameters based on the optimal replenishment policy of Example 1. One parameter is changed at a time, keeping the other parameters unchanged. The results are summarized in Table 1. Based on the numerical results, we obtain the following managerial implications

1. The total profit (TP) is highly sensitive with the change of the selling price $p$ and significantly sensitive with respect to the parameters P and $\lambda$. The retailer should give more concentration in selecting these parameters for taking the optimal decision. It is moderately sensitive with the change in the parameters $\varphi, d$ and $K$ and it is less sensitive with change in the parameters $\delta, \theta, M$ and $N$.
2. Maximum product amount at the very beginning of the cycle (S) is reasonably sensitive with the change of the parameters $K, \theta, \varphi, p$ and it is less sensitive for the parameters $\varphi, d, \lambda, P, \delta, M, N$
3. The ending inventory level (R) is reasonably sensitive with the change of the parameters $K, \theta, \varphi, p, M, N$ and slightly sensitive with the change of the all other parameters
4. From Table, we see that the increases in rate of deterioration $\theta$ leads to decrease in the total profit. Hence when the deterioration rate of products is more, the retailer should order less.
5. The backlogging rate decreases with increase in the backlogging parameter $\delta$. Hence, when

the backlogging rate decreases, the total profit decreases. To achieve maximum total profit , the retailer should increases the backlogging rate by ordering more quantity.

6. When the unit purchase cost increases the retailer's total profit per unit time decreases. Thus the retailers must reduce the unit purchase cost but negotiating with the supplier ensuring them that they are going to make a higher order size.
7. As the length of credit period M increases, the total optimal profit per unit time increases. This clearly suggests that if the permissible delay period increases, then it helps the retailer to prolong the payment to the supplier without penalty and earn more from the interest earned and eventually results in higher profit. To acquire more profit, the retailer have to concentrate on their credit period tactics and cycle length.
8. If the length of the period N increases, the total profit decreases slightly. Due to the increase in the trade credit period of the customer, measure of total profit for the retailer is slightly smaller

## 5. CONCLUSION:

In this work a EOQ model for instantaneous deteriorating items with selling price and stock linked hybridized market demand, discount facility for advanced prepayment and two level trade credit period is developed. The two-level trade credit policy is adopted which means the supplier offers the retailer a trade credit period and the retailer also offers credit period to the customer, which will lure more customers. Shortages are allowed which are partially backlogged. Backlogging rate is inversely proportional to the waiting time of the next replenishment. The concavity of the objective function of the formulated optimization model is explored mathematically by the Hessian matrix. Managerial insights are drawn from the sensitivity analysis carried out. From sensitivity analysis carried out the rate of change in the parameters $\varphi, K, p, h, \theta, \delta, \lambda, P, N$ and M is analyzed which helps to the business organization to make better managerial decisions. From the results obtained, the retailer can increase the total profit by offering 1) discount for prepayment facility which will motivate their customers to buy more goods 2) trade credit period for both retailer and customer which will lure more customers and consequently increase the profit. For future research, researchers can explore an extension of proposed model by taking into account some characteristics such as inflation, non-linear holding cost and two-warehouse inventory with partial trade credit period

## REFERENCES:


[1] S. Sana, K.S. Chaudhuri, A stock-review EOQ model with stock-dependent demand, quadratic deterioration rate, Advanced Modeling and Optimization 6 (2) (2004) 25–32.

[2] Z.T. Balkhi, L. Tadj, A generalized economic order quantity model with deteriorating items and time varying demand, deterioration, and costs, Int Trans Operational Res 15 (4) (2008) 509–517.

[3] S. Pal, G.S. Mahapatra, G.P. Samanta, A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness, Econ. Model. 46 (2015) 334–345.

[4] M.-A. Khan, A.A. Shaikh, G.C. Panda, I. Konstantaras, A.A. Taleizadeh, Inventory system with expiration date: Pricing and replenishment decisions, Comput. Ind. Eng. 132 (2019) 232–247.

[5] A.A. Shaikh, M.A. Khan, G.C. Panda, I. Konstantaras, Price discount facility in an EOQ model for deteriorating items with stock-dependent demand and partial backlogging, Intl. Trans. in Op. Res. 26 (4) (2019) 1365–1395

[6] A.X. Zhang, Optimal advance payment scheme involving fixed per-payment costs, Omega 24 (5) (1996) 577–582

[7] A.K. Maiti, M.K. Maiti, M. Maiti, Inventory model with stochastic lead-time and price dependent demand incorporating advance payment, Appl. Math. Model. 33 (5) (2009) 2433–2443.

[8] A. Thangam, Optimal price discounting and lot-sizing policies for perishable items in a supply chain under the advance payment scheme and two-echelon trade credits, Int. J. Prod. Econ. 139 (2) (2012) 459–472.

[9] Q. Zhang, Y.-C. Tsao, T.-H. Chen, Economic order quantity under advance payment, Appl. Math. Model. 38 (24) (2014) 5910–5921

[10] N.P. Zia, A.A. Taleizadeh, A lot-sizing model with back ordering under hybrid linked-to-order multiple advance payments and delayed payment, Transportation Research Part E: Logistics and Transportation Review 82 (2015) 19–37

[11] R. Li, Y.-L. Chan, C.-T. Chang, L.E. Ca´rdenas-Barro´n, Pricing and lot-sizing policies for perishable products with advance cash-credit payments by a discounted cash-flow analysis, Int. J. Prod. Econ. 193 (2017) 578–589

[12] M.A. Khan, A.A. Shaikh, G.C. Panda, I. Konstantaras, Two warehouse inventory model for deteriorating items with partial backlogging and advance payment scheme, RAIRO-Oper. Res. 53 (5) (2019) 1691–1708

[13] A.A. Shaikh, S.C. Das, A.K. Bhunia, G.C. Panda, M.A. Khan, A two-warehouse EOQ model with interval-valued inventory cost and advance payment for deteriorating item under particle swarm optimization, Soft Comput 23 (24) (2019) 13531–13546

[14] Y. Shi, Z. Zhang, S.-C. Chen, L.E. Ca´rdenas-Barro´n, K. Skouri, Optimal replenishment decisions for perishable products under cash, advance, and credit payments considering carbon tax regulations, Int. J. Prod. Econ. 223 (2020) 107514

[15] MS Rahman, MAA Khan, MA Halim, TA Nofal, Ali Akbar, Hybrid price and stock dependent inventory model for perishable goods with advance payment related discount facilities under preservation technology, Alexandria Engineering Journal (2021) 60, 3455-3465

[16] J.T. Teng, J. Min, Q. Pan, Economic order quantity model with trade credit financing for non-decreasing demand, Omega 40 (2012) 328–335.

[17] B. Sarkar, An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production, Appl. Math. Comput. 218 (2012) 8295–8308.

[18] J. Min, Y.W. Zhou, G.Q. Liu, S.D. Wang, An EPQ model for deteriorating items with

inventory–level-dependent demand and permissible delay in payments, Int. J. Syst. Sci. 43 (2012) 1039–1053.

[19] Teng, J.-T. On the economic order quantity under conditions of permissible delay in payments. J. Oper. Res. Soc. 2002, 53, 915–918

[20] Huang, Y.F. Optimal retailer's ordering policies in the EOQ model under trade credit financing. J. Oper. Res. Soc. 2003, 54, 1011–1015

[21] Teng, J.-T.; Goyal, S.K. Optimal ordering policies for a retailer in a supply chain with up-stream and down-stream trade credits. J. Oper. Res. Soc. 2007, 58, 1252–1255

[22] Teng, J.-T. Optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers. Int. J. Prod. Econ. 2009, 119, 415–423

[23] Kreng, V.B.; Tan, S.-J. Optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy. Expert Syst. Appl. 2011, 38, 9888–9899

[24] Teng, J.-T.; Lou, K.-R. Seller's optimal credit period and replenishment time in a supply chain with up-stream and down-stream trade credits. J. Glob. Optim. 2012, 53, 417–430

[25] Mahata, G. An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. Expert Syst. Appl. 2012, 39, 3537–3550

[26] Chung, K.-J.; Cárdenas-Barrón, L.E. The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade credit in the supply chain management. Appl. Math. Model. 2013, 37, 4653–4660

[27] Chung, K.-J.; Cárdenas-Barrón, L.E.; Ting, P.-S. An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit. Int. J. Prod. Econ. 2014, 155, 310–317

[28] Chen, S.-C.; Cárdenas-Barrón, L.E.; Teng, J.-T. Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity. Int. J. Prod. Econ. 2014, 155, 284–291

[29] Shah, N.H.; Cárdenas-Barrón, L.E. Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. Appl. Math. Comput. 2015, 259, 569–578

[30] Aliabadi, L.; Yazdanparast, R.; Nasiri, M.M. An inventory model for non-instantaneous deteriorating items with credit period and carbon emission sensitive demand: A signomial geometric programming approach. Int. J. Manag. Sci. Eng. Manag. 2019, 14, 124–136

[31] Liao, J.-J.; Srivastava, H.M.; Chung, K.-J.; Lee, S.-F.; Huang, K.-N.; Lin, S.-D. Inventory Models for Non-Instantaneous Deteriorating Items with Expiration Dates and Imperfect Quality under Hybrid Payment Policy in the Three-Level Supply Chain. Symmetry 2021, 13, 1695

[32] Shaikh, A.A.; Cárdenas-Barrón, L.E.; Manna, A.K.; Céspedes-Mota, A. An economic production quantity (EPQ) model for a deteriorating item with partial trade credit policy for price dependent demand under inflation and reliability. Yugosl. J. Oper. Res. 2020, 31, 139–151

[33] Abu Hashan Md Mashud, Md. Sharif Uddin, Shid Sankar Sana, A Two-Level Trade-Credit Approachto an Integrated Price-Sensitive Inventory Model with Shortages, Int. J. Appl. Comput. Math (2019) 5:121