

MATHEMATICAL MODELING AND BEHAVIOR ANALYSIS OF A JUICE PLANT

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Abstract: This study uses RPGT to analyses the behavior of a three-unit system and analyses system parameters. Since all of the units, A, B, and D, has parallel subcomponents, the system can still function even if one unit is operating at a reduced capacity. Due to the differing failure distributions across the three components, the system collapses when any one of them fails. RPGT is used to obtain expressions for System parameter values. To compare the failure and repair rates and their impact on the parameter values, graphs and tables are created. Additionally, profit optimization is covered.

Keywords: RPGT, Optimization, MTSF, Busy Period, Increasing failure and repair rates, Steady State.

1. Introduction:

Three unique units make up the system. The failure rates are exponentially distributed, and the rates of repair are universal, independent, and distinct for various operational units. Different units have different capacities. The fixes are flawless. When two units are in decreased condition or when any one of the units is in failed state, the system is down. Repair is separate from failure. All of the juicy fruits that are available in the area can be consumed by the locals or sold in the local market, but the famous of the area also want to make money off of their produce. As a result, juice is made from the excess quantity of juicy fruits and sold in local, national, and international markets where it is well-liked and in high demand due to its good taste. These plants have provided self-employment and job satisfaction. The juice plant can be operated both annually and continuously. Fruit of one variety or another is readily available. Each plant is capable of producing fruits that are already packaged and ready to eat. Fresh or frozen fruits are used to make juice, for which there are numerous crushing and pressing machines accessible in a facility. If one or more crushing or pressing units malfunction, the entire system operates at a reduced capacity. If there are more crushing or pressing units than necessary, one crushing unit will fail and the entire system will stop working.

Fresh fruit has a very short shelf life, so to extend it, the same preservative and other ingredients must be added in accordance with the specifications of a particular flavor. This process of adding ingredients and preservatives is known as processing, and in large juice plants, numerous processing units operate simultaneously. The efficiency of the processing unit, and consequently

that of the entire plant, is decreased when one or more but less than a specific level fail. According to customer/market need, the processed juice is placed in packaging of various sizes for marketing purposes. Again, in a large plant, these packing units are installed/mechanized in parallel, so the failure of one or more packing units will lower the working capacity of the packing system. As a result, the entire plant will suffer if the processed fruit is not packed right away because the juice left in the open will start to spoil at a higher rate and needs to be packed right away.

Kumar et al. (2018, 2017) have deliberated the behavior examination of a bread scheme and edible oil refinery plant. Asi et al. (2021) conducted a relative research of the five productive reliability methodologies. The major goal of this work, according to Kumar et al. (2019), is to examine and analyses a washing unit used in the paper industry using RPGT. Kumar et al. (2019) analyzed a cold standby agenda with priority for PM covers two identical subunits by server failure utilizing RPGT. Gupta (2008), Chaudhary et al. (2013), Goyal and Goel (2015), Yusuf (2012) and Gupta et al. (2016) have discussed conduct with perfect and imperfect switch-over of arrangements by various techniques. Figure 1 below depicts a steady state transition diagram taking these transition rates into account. Transition probabilities, sojourn periods, state probabilities, and reliability metrics are modeled using RPGT expressions. Each instance is discussed after preparing sensitivity tables that modify the other variables and keep either failure or repair rats of units.

2. Assumptions and notations

- Failure/Repair rates are constant.
- A, B, and C remain used aimed at working state.
- a, b and c are used for unsuccessful state.

3. Transition Diagram: -

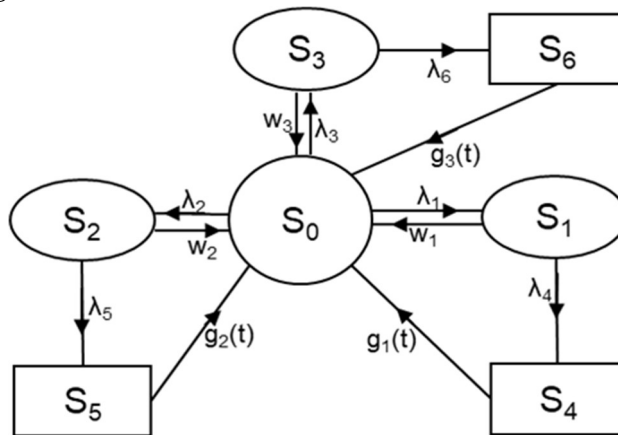


Figure1: Transition Diagram

$S_0 = ABC$
 $S_3 = ABC\bar{C}$

$S_1 = \bar{A}Bc$
 $S_4 = aBC$

$S_2 = A\bar{B}c$
 $S_5 = AbC$

$$S_6 = ABc$$

Probability Density Function: -

$$q_{i,j}^{(t)}$$

$$q_{0,1} = \lambda_1 e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}$$

$$q_{0,2} = \lambda_2 e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}$$

$$q_{0,3} = \lambda_3 e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}$$

$$q_{1,0} = w_1 e^{-(w_1 + \lambda_4)t}$$

$$q_{1,4} = \lambda_4 e^{-(w_1 + \lambda_4)t}$$

$$q_{2,0} = w_2 e^{-(w_2 + \lambda_5)t}$$

$$q_{2,5} = \lambda_5 e^{-(w_2 + \lambda_5)t}$$

$$q_{3,0} = w_3 e^{-(w_3 + \lambda_6)t}$$

$$q_{3,6} = \lambda_6 e^{-(w_3 + \lambda_6)t}$$

$$q_{4,0} = g_1(t)$$

$$q_{5,0} = g_2(t)$$

$$q_{6,0} = g_3(t)$$

Cumulative density functions in moving from state 'i' to state 'j' by taking Laplace Transform of above function for infinite time interval is given as under: -

$$P_{ij} = q_{i,j}^{*(t)}$$

$$p_{0,1} = \lambda_1 / (\lambda_1 + \lambda_2 + \lambda_3)$$

$$p_{0,2} = \lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3)$$

$$p_{0,3} = \lambda_3 / (\lambda_1 + \lambda_2 + \lambda_3)$$

$$p_{1,0} = w_1 / (w_1 + \lambda_4)$$

$$p_{1,4} = \lambda_4 / (w_1 + \lambda_4)$$

$$p_{2,0} = w_2 / (w_2 + \lambda_5)$$

$$p_{2,5} = \lambda_5 / (w_2 + \lambda_5)$$

$$p_{3,0} = w_3 / (w_3 + \lambda_6)$$

$$p_{3,6} = \lambda_6 / (w_3 + \lambda_6)$$

$$p_{4,0} = g_1^*(0) = 1$$

$$p_{5,0} = g_2^*(0) = 1$$

$$p_{6,0} = g_3^*(0) = 1$$

Probability density functions for Mean Sojourn Times

$$R_i(t)$$

$$R_0^{(t)} = e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}$$

$$R_1^{(t)} = e^{-(w_1 + \lambda_4)t}$$

$$R_2^{(t)} = e^{-(w_2 + \lambda_5)t}$$

$$R_3^{(t)} = e^{-(w_3 + \lambda_6)t}$$

$$R_4^{(t)} = \overline{G_1(t)}$$

$$R_5^{(t)} = \overline{G_2(t)}$$

$$R_6^{(t)} = \overline{G_3(t)}$$

Value of the parameter μ_i giving Mean Sojourn Times

$$\mu_i = R_i^*(0)$$

$$\mu_0 = 1/(\lambda_1 + \lambda_2 + \lambda_3)$$

$$\mu_1 = 1/(w_1 + \lambda_4)$$

$$\mu_2 = 1/(w_2 + \lambda_5)$$

$$\mu_3 = 1/(w_3 + \lambda_6)$$

$$\mu_4 = -g_1^*(0)$$

$$\mu_5 = -g_2^*(0)$$

$$\mu_6 = -g_3^*(0)$$

Transition Probabilities from the initial state ‘0’

$$V_{0,0} = 1 \text{ (Verified)}$$

$$V_{0,1} = (\lambda_1/\lambda_1 + \lambda_2 + \lambda_3)$$

$$V_{0,2} = (\lambda_2/\lambda_1 + \lambda_2 + \lambda_3)$$

$$V_{0,3} = \dots\dots\text{Continuous}$$

4. Results:

MTSF(T_0): The states to which the system move from initial state ‘0’, before going to any failed state are: ‘i’ = 0,1,2,3 taking ‘ ξ ’ = ‘0’.

$$MTSF (T_0) = \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr(sff)} i \right) \right\} \mu_i}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[1 - \sum_{sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr(sff)} \xi \right) \right\}}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right]$$

Availability of the System: The states in which the system works in reduced or full are ‘j’ = 0,1,2,3, taking ‘ ξ ’ = ‘0’ the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow j}) \right\} f_j, \mu_j}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right]$$

$$A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j \right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1 \right]$$

Busy Period of the Server: The regenerative states where server ‘j’ = 1,2,3,4,5,6 and regenerative states are ‘i’ = 0 to 6, taking $\xi = ‘0’$, the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow j}) \right\} n_j}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right]$$

$$B_0 = \left[\sum_j V_{\xi,j}, n_j \right] \div \left[\sum_i V_{\xi,i}, \mu_i^1 \right]$$

Expected Fractional Number of Inspections by the repair man: The regenerative states where the repair man do this job $j = 1,2,3$ Taking ‘ ξ ’ = ‘0’, the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,SR} \left\{ \frac{\{pr(\xi^{SR \rightarrow j})\}}{\prod_{k_1 \neq \xi} \{1 - V_{k_1 k_1}\}} \right\} \right] \div \left[\sum_{i,SR} \left\{ \frac{\{pr(\xi^{SR \rightarrow i})\} \mu_i^1}{\prod_{k_2 \neq \xi} \{1 - V_{k_2 k_2}\}} \right\} \right]$$

$$V_0 = [\sum_j V_{\xi,j}] \div [\sum_i V_{\xi,i}, \mu_i^1]$$

5. Particular Cases

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda$

$w_1 = w_2 = w_3 = w$ say

Availability of the System (A₀) Table

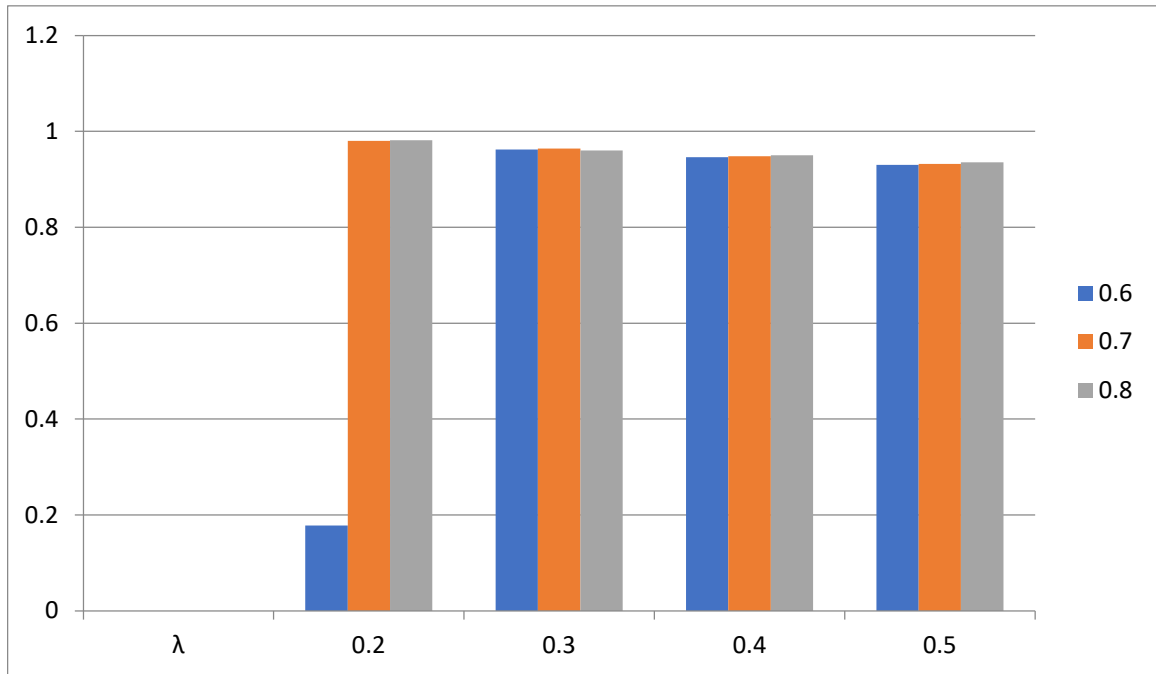


Figure 2: Availability of the System Graph

Busy Period of the Server (B₀)

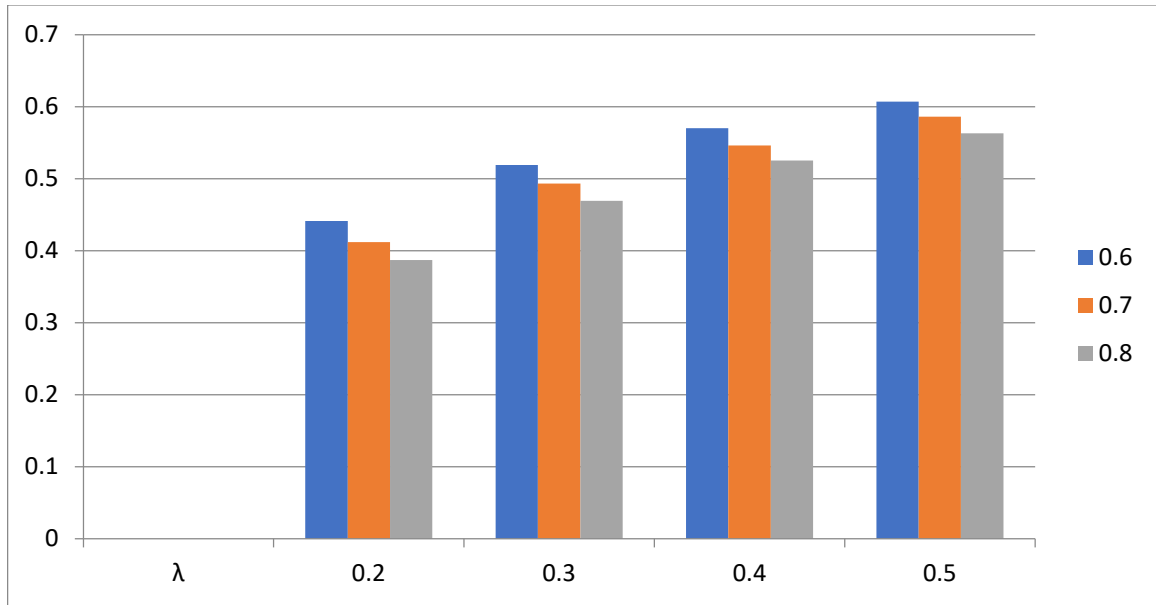


Figure 3: Busy Period of the Server Graph

Expected Number of Server's Visits (V_0)

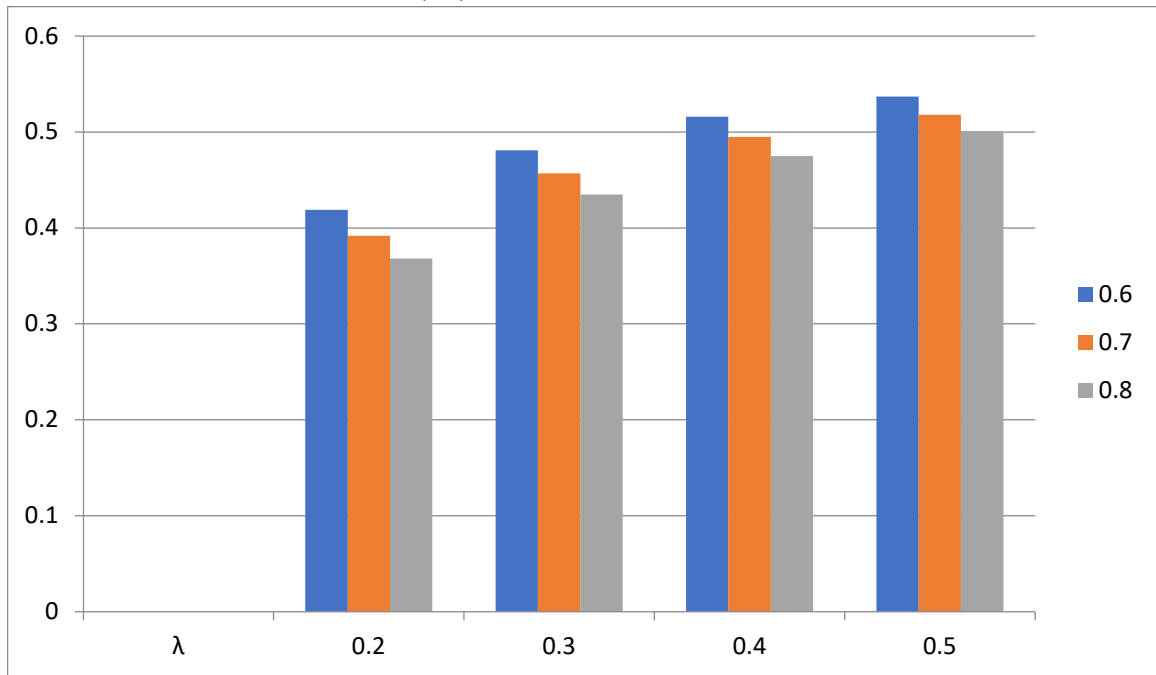


Figure 4: Expected Number of Server's Visits Graph

6. Conclusion

From the analytical and figure discussions, it is noted availability of the system, profit function, expected number of inspections by repairman are decremented with increase failure rate and they all increase as the repair rate increases. Busy period of server and mean time to system failure are

decremented with increase repair rates. The effectiveness and the reliability of the plant can be improved by increasing repair rate and decreasing the failure rate.

7. References

1. Asi, J. J., Seghier, M. E. A. B., Ohadi, S., Dong, Y. and Plevris, V. (2021). A Comparative Study on the Efficiency of Reliability Methods for the Probabilistic Analysis of Local Scour at a Bridge Pier in Clay-Sand-Mixed Sediments. *MDPI*, 2, 63-77.
2. Kumar, A. Garg, D. and Goel, P. (2019). Mathematical modeling and behavioral analysis of a washing unit in paper mill. *International Journal of System Assurance Engineering and Management*, 10, 1639-1645.
3. Kumar, A. Garg, D. and Goel, P. (2019). Sensitivity Analysis of a Cold Standby System with Priority for Preventive Maintenance. *Journal of Advances and Scholarly Researches in Allied Education*, 16(4), 253-258.
4. Kumar, A., Goel, P. and Garg, D. (2018). Behaviour analysis of a bread making system. *International Journal of Statistics and Applied Mathematics*, 3(6), 56-61.
5. Kumar, A. Garg, D. and Goel, P. (2017). Mathematical modeling and profit analysis of an edible oil refinery industry. *Airo International Research Journal*, XIII, 1-14.
6. Gupta, R. Sharma, S. and Bhardwaj, P. (2016). Cost Benefit Analysis of a Urea Fertilizer Manufacturing System Model. *Journal of Statistics a Application & Probability Letters An International Journal*, 3, 119-132.
7. Goyal, V. and Goel, P. (2015). Behavioral Analysis of Two Unit System with Preventive Maintenance and Degradation in One Unit after Complete Failure Using RPGT, 4, 190-197.
8. Nidhi, C. Goel, P. and Garg. (2013). Availability Analysis of a Soap Industry Using Regenerative Point Graphical Technique (RPGT). *IJIFR*, 1, 2347-1697.
9. Yusuf, I. (2012). Availability and Profit Analysis of 3-out-of-4 Repairable System with Preventive Maintenance. *International Journal of Applied Mathematical Research*, 1 (4), 510-519.
10. Gupta, V. K. (2008). Behavior and profit analysis of some process industries. Ph.D. Thesis, NIT Kurukshetra (India), (2008).