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AN INVENTORY MODEL TO MINIMIZING THE TOTAL COST WITH TIME–DEPENDENT DEMANDS AND TIME–VARYING HOLDING COST UNDER PARTIAL BACKLOGGING

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Abstract: A deterministic inventory model with time – dependent demand and time – varying holding cost where deteriorating is time proportional. The model is solved analytically by minimizing the total inventory cost. In contrast to deteriorating, amelioration refers to a situation where stocked items incur increased value, quantity, or utility while in stock. It is generally seen in poultry, piggeries, wine industries etc. When these items are kept in the farm or the sales counter, they usually incur increase in quantity and value. In this research paper, we study an inventory model that outlines the optimal replenishment decision for ameliorating items with a partially backlogged time- varying demand rate to raise productivity and understand opportunity cost focused essentially on deteriorating inventory, giving little or no attention to its ameliorative nature. The proposed model based on the global market strategies as for how the demand varies of the new seasonal products when they entered in the markets. The model has developed for the seasonal products or new consumer goods. The demand rate has considered Ramp- type based on the seasonal products having a time – dependent deteriorating rate.

Key words: *Inventory Model; Deterministic; Shortages; Fractional Polynomial; Component; Deteriorating items; shortage; time varying holding cost; Ameliorating inventory; Replenishment decision; partial backlogging; lost sales.*

Introduction: Inventory is fundamentally a product that requirements to be well-ordered in demand to maximize earnings. Inventory controllers include approximately responsibilities like delivery, procuring, container, tracking and many more facilities. In Inventory contains properties which are mainly of two categories: firstly, goods are which declining with interval alike Bread, Milk, fruit etc. and the next type contains the assets which do not decline with time if located in a comforting situation like a paper gold-leaf silver etc. The expected life phase of the primary caring of objects is low-slung and further kind of objects has a arcade of petite life. Numerous investigators have measured the mandate rate to be continuous and some careful the due amount to be a rectilinear or quadratic occupation of period. The request rate of confident goods was originated to be dependent on export price and some dependent on period. In the bazaar, we usually detect that the usefulness of approximately item intensifications with retro. For occurrence, in breweries, the assessment of some carried violet growths with time. In farmhouses, the measure of fish, fast-growing animals counting broilers, conformist and so on, growths with interval. These phenomena are labelled upgrading. In the out-dated inventory models, unique of the expectations was that the objects conserved their corporeal appearances although they were reserved warehoused in the account. This assumption is obviously true for maximum items, but not for all. Though, the declining items are subject to a continuous damage in their crowds or usefulness through their generation due to decay, injury, decay and fine of other reasons. Decline is defined as deterioration or destruction such that the element cannot use for its unique determinations. The consequence of declining is actual significant in many inventory classifications. Food items, Narcotics and harmful ingredient are sample of substances in which adequate deteriorating can revenue place throughout the usual storage retro of the components and accordingly this damage must be occupied into version when investigating the classification, as declining of an item and holding cost of inventory be contingent upon the time. Declining rating substances means that the item that developed damaged or missing their bordering value finished time. Manpower and apparatuses can be used intelligently to shrink the block of speculation on them, but in non-appearance of raw resources and well-appointed goods in their storing, the arrived worth and the marketplace organization may breakdown. Though, possession inventory designates a obstructive of speculation without any turnover. The price of resounding, impairment to the stock throughout stowage and depression of the typical affect the profit apparatus of the innovativeness meaningfully. Consequently, appropriate inventory organization is a deplorable essential of the period so that the revenue restrictions of the innovativeness should not be pretentious. Declining is the greatest significant influence in any inventory organization that cannot be discounted in the investigation. This characteristic of the products generates unlimited interest amongst provider's constructors, merchants as it transports the effectiveness side by side of the goods downhearted to nil due to deterioration, impairment, evaporations, or damage of value ended time. Different types of food merchandises like fruits, vegetables etc. it is observed that the inventory models developed under the assumption of a lifetime of a product is endless until its storage i.e., an item once stowed in a storeroom remains unmoved and wholly serviceable for sufficient upcoming demand. However, in realism, it is not accurate due to the consequence of declining in the conservancy of frequently used fleshly belongings like wheat, paddy, or any additional kind of food grains vegetables, fruits, drugs, etc. if the request is more than inventory or objects in stock; it leads to absences or stock out. Shortages can be characterized hooked on two altered methods, primary case is recognized as spinal order or backlogging in which request is not achieved by the merchants. For example: if a purchaser goes to purchase some accounts and if accounts are not accessible at that interval, then he can delay buying for approximately period and he depend upon the storekeeper to deliver the accounts as soon as conceivable. Hence, deficiencies are allowed with fractional backlogging case. The additional one is called missing sales situation where the request is totally lost. For example: electronic apparatus and devices etc. have identical diminutive life span owing to ground-breaking variations in the reproductions and knowledge. In perform consumer may be ready to delay acquisition of approximately substances consumer will not postponement and buying them from another foundations.

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Assumption: The fundamental assumptions and notation, used in the proposed model are given as follows:

1. The replenishment occurs instantaneously, i.e., lead time is negligible.
2. The deteriorating rate function Θ(t) is considered as the time - dependent deterioration rate defined as Θ(t) = at, for t > 0 and 0 < a < < 1.
3. With above assumption we propose two model as below:
   I. In model 1, shortages are allowed, that is completely backlogged.
   II. In model 2, shortages are allowed, that is partially backlogged.

Let us assume b(t) be the fraction where t is the waiting time up to the next replenishment.

We consider b(t) = $\frac{1}{1+\tau}$, where τ is known as the backlogging parameter as positive constant.

4. The demand rate D_t is assumed to be a ramp – type function of time,

$D_t = D_0$[t-(t-η) H(t-η)], $D_0 > 0$;

Where, H(t – η) is defined as follows:

$$H(t-\eta) = \begin{Bmatrix} 1\ ;\ t \geq \eta \\ 0\ ;\ t \leq \eta \end{Bmatrix}$$

Notation:

D_0	: Demand Rate.
S_l	**:** The initial inventory levels.
τ	: Backlogging parameter and $\tau > 0$.
$L(t-\eta)$	: The Heaviside's function.
T	: The fixed length of each production cycle.
C_a	: Inventory holding cost per unit.
C_s	: Shortage cost per unit.
C_c	: The purchasing cost per unit time.
C_l	: The cost of lost sale per unit time.
I_1	: The inventory level time at time $[0,\eta]$
I_2	: The inventory level at time $[\eta, t_1]$
I_3	: The inventory level at time $[t_1, T]$
P	: The total amount of inventory produced and purchased.
P^*	: Optimum value of P.
S^*	: Optimum value of S.
$TC_a(t_1)$	: Average total cost per unit time for model 1.
$TC_s(t_1)$	: Average total cost per unit time for model 2.

Mathematical Formulation and Solution, Model 1: For completely backlogging shortage.

In the projected model, we have expected that S > 0 is primary record level. Record level will be reduced owing to request and decline amount throughout the period intermission [0, η] and touched nil level at the time interval [η, t_1]. Shortage will transpire throughout the time-period [t_1,T], which is measured as finally backlogged see in below:

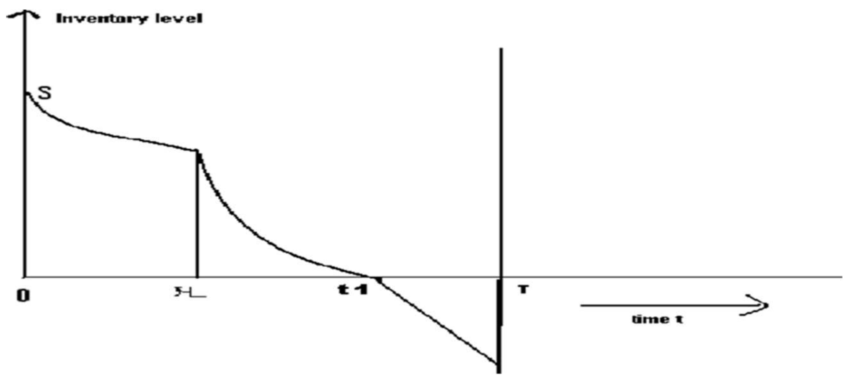

Figure 1: An EOQ model of Ramp –Type demand with complete backlogging.

Throughout the time [0,η], the inventory reduces owing to the decline and request both. Hence, the record level at any time throughout [0,η] is designated by the differential equation, given as below:

$$\frac{dI_1(t)}{dt} + atI_1(t) = -D_0 t, \qquad 0 \le t \le \eta \qquad (1)$$

During $[\eta, t_1]$ inventory reduces due to the decline and request and spreads to zero level.

Hence, the differential equation is given as follows:

$$\frac{dI_2(t)}{dt} + atI_2(t) = -D_0 \eta, \qquad \eta \le t \le t_1 \qquad (2)$$

With the limit condition $I_2(t_1) = 0$ and throughout the time interval $[t_1, T]$ shortages will be arising which is totally backlogged defined by the differential equation

$$\frac{dI_3(t)}{dt} = -D_0 \eta \qquad t_1 \le t \le T \qquad (3)$$

When 0 < a < 1, we ignore the higher power of a with the limit condition $I_3(t_1) = 0$. Using the situations $I_1(0) = S$, $I_2(t_1) = 0$ and $I_3(t_1) = 0$, the solution of equation (1), (2) and (3) will be given by

$$I_1 = \frac{D_0}{a}\left(e^{\frac{-at^2}{2}} - 1\right) + Se^{\frac{-at^2}{2}} \quad \text{for } 0 \le t \le \eta \qquad (4)$$

For solving Equation (2), we use our assumption that a << 1, then by taylor series expansion we have.

$$e^{\frac{-at^2}{2}} = 1 + \left(\frac{-at^2}{2}\right) + \frac{1}{2!}\left(\frac{-at^2}{2}\right)^2 + \dots\dots\dots$$

Neglecting the higher power of a we get, $e^{\frac{-at^2}{2}} = 1 - \frac{1}{2}at^2$. Then, the solution of Equation (2) becomes.

$$I_2 = D_0 \eta a e^{\frac{-at^2}{2}} [(t_1 - t) + \frac{(t_1^3 - t^3)}{6}] \quad \text{for } \eta \le t \le t_1 \tag{5}$$

The solution of equation (3) is given as below:

$$I_3 = - D_0 \eta (t - t_1) \quad \text{for } t_1 \le t \le T \tag{6}$$

Next, to find the maximum inventory level we use the condition as $I_1(t_1) = 0$. Then, we get the value of maximum inventory level given as:

$$I_{max} = S = \frac{D_0}{a} (e^{\frac{at_1^2}{2}} - 1)$$

(7)

Then, Equation (4) becomes.

$$I_1(t) = \frac{D_0}{a} (e^{\frac{a(t_1^2 - t^2)}{2}} -1)$$

(8)

Therefore, the total amount of deteriorated units is

$$D = S - \int_0^{t_1} D(t) dt$$

$$= S - [\int_0^{\eta} D_0 \, t \, dt + \int_{\eta}^{t_1} D_0 \eta \, dt]$$

Putting the value of S, in the above equation we have

$$D = \frac{D_0}{a}(e^{\frac{at^2}{2}} - 1) - \frac{D_0 \eta^2}{2} - D_0 \eta (t_1 - \eta) \tag{9}$$

Therefore, the average total cost per unit time is given by

$$TC_a(t_1) = \frac{C_c D}{T} + \frac{C_a}{T} \int_0^{t_1} I(t) dt - \frac{C_s}{T} \int_{t_1}^{T} I(t) \, dt$$

$$TC_a(t_1) = \frac{C_c D}{T} + \frac{C_a}{T} [\int_0^{\eta} I(t) \, dt + \int_{\eta}^{t_1} I_2 (t) \, dt] - \frac{C_s}{T} \int_{t_1}^{T} I_3(t) \, dt \tag{10}$$

Now, substituting the value of D from equation (9) and I(t) given by equation (4), (5) and (6) the value of S from Equation (7) in the above equation we have

$$TC_a(t_1) = \frac{C_c}{T}[\frac{D_0}{a}(e^{\frac{at_1^2}{2}} - 1) - \frac{1}{2} D_0 \eta^2 - D_0 \eta \, (t_1 - \eta) + \frac{C_a}{T}\{ -\frac{1}{6} D_0 \eta^3 + \frac{1}{2} D_0 t_1^2 \eta - \frac{1}{72} D_0 \eta a (t_1^6 - \eta^6) + \frac{1}{4}(-\frac{1}{6} D_0 \eta - \frac{1}{2} D_0 \eta a)(t_1^4 - \eta^4) + \frac{1}{6} D_0 \eta a (t_1 + \frac{1}{6} t_1^3)(t_1^3 - \eta^3) - \frac{1}{2} D_0 \eta (t_1^2 - \eta^2) + D_0 \eta (t_1 + \frac{1}{6} t_1^3)(t_1 - \eta)\} - \frac{C_c}{T}(-\frac{1}{2} D_0 \eta (T^2 - t_1^2) + D_0 \eta t_1 (T - t_1))]$$

(11)

The necessary condition for minimization of the average cost $TC_a(t_1)$ is is $\frac{dTC_1(t_1)}{dt_1} = 0$. Let $j(t_1) = \frac{dTC_1(t_1)}{dt_1} = 0$. Then the above equation yields the equation.

$j(t_1)= \frac{C_3}{T}\{D_0 t_1(e^{\frac{at_1^2}{2}}$ - $D_0\eta)$ + $\frac{C_1}{T}$(- $\frac{1}{12}D_0\eta a t_1^5$ + (- $\frac{1}{6}D_0\eta$ - $\frac{1}{2}D_0\eta a)t_1^3$ + $\frac{1}{6}D_0\eta a(1+\frac{1}{2}t_1^2)(t_1^3 - \eta^3)$ + $\frac{1}{2}D_0\eta a(t_1+\frac{1}{6}t_1^3)t_1^2$ + $D_0\eta(1+t_1^2)(t_1$-η) + $D_0\eta(t_1+\frac{1}{6}t_1^3)\}$ - $\frac{C_2}{T}D_0\eta$(T-t_1)
(12)

Again, we consider that t_1 = 0 then we obtain the value of j(0) as :

$$j(0) = - \frac{D_0\eta}{6T}(6C_3 + C_1\eta^3 a + 6C_1\eta + 6C_2 T) < 0 \quad (13)$$

Now, it is clear that j(0) < 0. Again, we substitute the value t_1= T, then we have

j_1(T) = $\frac{D_0}{12T}\{12C_3(Te^{\frac{aT^2}{2}}$ - η) + $C_1\eta a T^5$ + $2C_1\eta T^3$(3+a) - $C_1\eta^4$a(2+T^2) + $24C_1\eta$T-$6C_1\eta^2(2+T^2)\}$
(14)

Next, take second-order differential equation of T$C_1(t_1)$ and set $\frac{d^2TC_1(t_1)}{dt_1^2} = f_1(t_1)$. Thus, we have

$$f_1(t_1)=\frac{D_0}{12T}\{12C_3 e^{\frac{aT^2}{2}}(1+at_1^2)+5C_1\eta a t_1^4+6C_1\eta t_1^2(3+a)-2C_1\eta^2 t_1(a\eta^2+6)+12\eta(2C_1+C_2)\}>0 \quad (15)$$

By our assumption, it is clear that η< T and a < 1.Since, as the power of η increases the value of η decreases, i.e., η > η^2 > η^3 >… and the value of $e^{\frac{aT^2}{2}}$ >1. So, the above equation $f_1(t_1)$ > 0 and it implies that, $f_1(t_1)$ is a strictly monotone increasing function and and equation (12) has a unique solution $t_1 = t_1^* \epsilon$ (0, T).

Substituting $t_1 = t_1^*$ in the equation (7), we find that the optimum value of S is given by

$$S^* = \frac{D_0}{a}\{e^{\frac{(at_1^*)^2}{2}} - 1). \quad (16)$$

And the optimum value of P is therefore of Q is therefore given by.

$$P^* = S^* + D_0\eta(T-t_1^*)$$
$$= \frac{D_0}{a}\{e^{\frac{(at_1^*)^2}{2}} - 1) + D_0\eta(T-t_1^*) \quad (17)$$

And the minimum value of the average total cost T$C_1(t_1)$ is thus T$C_1(t_1^*)$.

Sensitivity Analysis of Model 1: Sensitivity investigation is helps to regulate how" sensitivity" a model is conferring to variations in the standards of the limitations of the model and to variations in the construction of the model. In this paper, we can see the sensitivity of the optimum result of our examples to fluctuation the morals of altered limitations connected with the model. The sensitivity analysis is achieved by altering each of the limitations C_a, C_s, C_c, D_0 and η, a by -50%, -25%, 25%, 50% taking one limitation at a time and possession the residual limitations unaffected. The consequence is obtainable in Table 1. On the foundation of consequences shown in Table 1, we can representation the following points as:

- TC_a^* has high sensitivity if we revolution the limitations C_s, D_0 and η while judicious sensitivity to changes in C_a andC_c,C_s, D_0 and η limitation are more pretentious our projected model.
- t_1^* has high sensitivity if we modification the limitations C_s, C_c and η at the same time reasonable sensitivity to alteration the limitation C_a at the identical time as unresponsive to changes in D_0 accordingly the parameter D_0 is constant in all process of our proposed inventory model.
- S^* Has high sensitivity if we change the parameters C_s and η even as moderately sensitive to changes in the parameterC_a, while intensely sensitive to changes in the parameters D_0 and η.

Parameter	**PCPV**	t_1^* **%change in**	S^* **%change in**	P^* **%change in**	C^***%change in**
C_a	-50	9.085	8.43	2.11	-2.507
	-25	2.004	4.049	1.009	-1.22
	25	-4.001	-3.847	-0.948	1.165
	50	-7.443	-7.443	-1.824	2.276
C_c	-50	-28.21	-48.47	-10.85	-43.10
	-25	-13.19	-24.65	-5.860	-20.18
	25	11.655	24.67	6.317	17.92
	50	22.07	49.05	12.87	33.93
C_s	-50	71.53	0.060	19.13	-14.39
	-25	26.40	0.024	6.772	-5.776
	25	-17.14	-0.036	-4.133	4.1379
	50	-28.93	-0.060	-6.814	7.246
D_0	-50	0	0	-30.32	-49.99
	-25	0	0	-15.16	-24.99
	25	0	0	15.16	24.99
	50	0	0	30.32	50.00
η	-50	-40.39	-64.47	-49.81	-36.77
	-25	-18.47	-33.53	-24.32	-15.99
	25	15.72	33.93	22.79	12.35

	50	29.35	67.35	44.01	21.87

Thus, the parameters D_0 and η are more affected the proposed model.

- P^* has low sensitivity if we change the parameters C_a, as highly sensitivity to changes the parameters D_0 and η and lowly sensitive to change the parameters $\boldsymbol{C_c}$ and $\boldsymbol{C_s}$. Thus, the parametersD_0, η are more affected by our model, so use them in our proposed model be careful.

Parameter	**PCPV (%)**	t_1^* **% change in**	S^* **%change in**	P^* **%change in**	C^* **% change in**
C_a	-50	4.955	10.16	6.956	-2.82
	-25	2.411	4.88	3.341	-1.37
	25	-2.27	-4.507	-3.08	1.29
	50	-4.44	-8.700	-5.944	2.530
C_s	-50	-5.00	-9.754	-6.664	-5.59
	-25	-2.477	-4.89	-3.347	-2.75
	25	0.145	4.929	3.372	2.676
	50	4.823	9.88	6.76	5.27
C_c	-50	38.34	91.47	62.86	-19.85
	-25	15.79	34.11	23.390	-7.861
	25	-11.32	-21.37	-14.58	5.467
	50	-19.73	-35.58	-24.22	9.464
C_d	-50	-28.69	-49.16	-33.37	-33.95
	-25	-13.73	-25.59	-17.45	-15.68
	25	12.47	26.53	18.182	13.39
	50	23.69	53.04	36.40	24.79
D_0	-50	0	-50	-50	-49.99
	-25	0	-25	-25	-24.99
	25	0	25	24.99	25.00
	50	0	50	50	50.00
η	-50	-43.23	-67.78	-60.86	-36.88
	-25	-19.97	-35.97	-31.42	-15.77
	25	16.90	36.68	30.96	11.51
	50	31.06	71.83	59.93	19.66
τ	-50	-7.035	-13.58	-1.82	-14.15
	-25	-2.853	-5.62	-0.80	-5.96
	25	2.101	4.250	0.63	4.607
	50	3.71	7.57	1.141	8.30

Table 2: Effect of changes in the parameters

Sensitivity Analysis of model 2

- TC_2^*, has extremely sensitive if we change the limitationsC_d, D_0 and η while temperately sensitive to variations inC_a, C_s, C_c and τ. Thus, the variations in limitations C_d, D_0 and η are more pretentious our anticipated inventory model.
- t_1^* has humble sensitivity if we variation the limitations C_a, C_s, and τ at the similar time as insensitive to variations in D_0 while has extremely sensitivity if we variation the limitations C_c, C_d and η. Thus, the variations in limitationsC_c, C_d and η are more affected our projected inventory model.
- S^* has temperate sensitivity if we variation the limitations C_a, C_s and τ, uniform as extremely sensitive to variations in the limitation C_c, C_e, D_0 and η. Thus, the variations of limitationsC_c, C_e, D_0 and η are more pretentious our planned inventory model.
- Q^* has reasonable sensitivity if we change the limitations C_a, C_s, τ uniform as highly sensitive to changes in the parameters C_c, C_e, D_0 and η. Thus, the changes of parameterC_c, C_e, D_0 and η are more pretentious our proposed inventory model.

Conclusion: From the above observation, we determine that it is feasible to determination two EOQ models for Incline type mandate rate with time – dependent decline rate. The primary model, in which storage is allowable, that is whole backlogged and another model, in which deficiency is allowable for a suitable division of request which is moderately backlogged. In the record of the models, the authors measured their model with a continuous deterioration amount. But, in actual life condition, items may be declined, i.e., deterioration rate is comparative with time and the determined generation can be organized by the manufacture system, i.e., the manufacturer can fix the extreme lifetime of the invention.

In the consequence, we encompass the inventory models for deteriorating items with ramp – type demand rate in several ways, given as follow.

1. In the suggested model, we use time – dependent deterioration rate.
2. In the suggested model, we allow shortages which are totally backlogged and partial backlogging with time – dependent deterioration rate.
3. The suggested model is solved analytically to obtain the optimal solution, sensitivity analysis and discussed.

Also, the proposed model can assist the manager to determine accurately the optimal order quantity and average total cost per unit. Moreover, the proposed model can be used in inventory control of certain deteriorating items such as food items, electronics, components, fashionable commodities etc. in future work, and it is also possible to incorporate realistic assumption such as probabilities demand as a finite rate of replenishment in the proposed model.

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