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NOT DISTRIBUTIVE LATTICE OVER AN L[1] $\mathcal{C}(L)$– SPACE

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ABSTRACT

J. Lindenstrauss & H. P. Rosenthal in [1] ***The $\mathfrak{L}_p$ – Spaces*** *have claimed that there are sufficiently many Boolean algebras of projections.* ***'Classification of spatial $\mathfrak{L}^p$ AF algebras***, *[2] by N. Christopher Phillips and Maria Grazia Viola have discussed the specific class of operator algebras.* ***The duo again discussed*** *Cuntz algebras in [3] on a commutative Euclidean ring in which every non zero element possess multiplicative inverse forming a field ω. The n+1 copies of ω whose direct sum and scalar product operations can help to construct a polynomial ring having polynomials of degree less than or equal to n that also can be verified as the metric space by suitably constructing the distance function d as the metric. Sergio A. Alwarez [4] has given arithmetic on* ***$\mathfrak{L}_p$ – Spaces*** *that promoted the Minkowski inequality including the definitions in the introduction chapter followed. Also it can be verified as a linear space of dimension n +1. A metric space upon which the function norm is introduced and taking the collection of convergent sequences over it forms a [2] Banach algebra$\mathfrak{B}$/Banach space or [1] an L^p- space. On constructing bases for ideals in polynomial rings over the integers [6], Ayoub C.W. claimed that Suitably defining the partial order, join and meet operations upon it, the smallest σ- algebra containing $\mathfrak{B}$ is $\mathfrak{B}^*$ will be a lattice. Using the inequality of the sum of the determinants greater than the determinant of the sum, the distributive property is shown to be failed in this lattice.*

Keywords: *Banach space, p^{th} Norm, Banach Algebra, Ring, Vector space, Linear Space, Metric Space, Abelian, Vector, continuity of a linear functional, determinant of the matrix, supremum norm.*

1. INTRODUCTION

An algebra $(\omega, +, \cdot)$is a field if the binary operations + and are closed, associative in ω and zero element, unity are in ω, distributive laws hold in ω, all non-zero elements in ω are units while all nonzero elements in ω possess inverses with respect to +, abelian and commutative properties hold.

Let $\omega \times \omega \times \ldots \times \omega = \omega^{n+1} = \{(a_0, a_1, a_2, \ldots, a_n) | a_i \in \omega, 0 \leq i \leq n\}$ be the Cartesian product constructed over the field ω. (1.1)

Also, $\omega[x]$ is a polynomial ring. (1.2)

With respect to the component wise addition $\oplus$ on ω^{n+1}defined by

$$(a_0, a_1, a_2, \ldots, a_n) \oplus (b_0, b_1, b_2, \ldots, b_n) = (a_0 + b_0, a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n) \quad (1.3)$$

The component wise multiplication or scalar product $\odot$ on ω^{n+1}defined by

$$(a_0, a_1, a_2, \ldots, a_n) \odot (b_0, b_1, b_2, \ldots, b_n) = (a_0 \cdot b_0, a_1 \cdot b_1, a_2 \cdot b_2, \ldots, a_n \cdot b_n) \quad (1.4)$$

Taking the coefficients from ω, $\bar{f} \in \omega^{n+1}[x]$ appears to be $\bar{f} = (a_0\ a_1\ a_2 \ldots\ a_n)\bar{x}^T$

$= (a_0 a_1\ a_2 \ldots\ a_n) \begin{pmatrix} 1 \\ x \\ x^2 \\ \vdots \\ x^n \end{pmatrix}$ Where $(a_0 a_1\ a_2 \ldots\ a_n) \equiv (a_0, a_1, a_2, \ldots, a_n)$ a string formed by concatenation of the components of the vector $(a_0, a_1, a_2, \ldots, a_n) \in \omega^{n+1}$

So, $\bar{f} = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \in \omega^{n+1}[\bar{x}]$

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Let us see $(\omega^{n+1}[x],\oplus,\odot)$ is a ring of polynomials of degree $\leq n$.

Definition: $\omega^{n+1}[x]=\{\bar{f}=a_0+a_1x+a_2x^2+\cdots+a_nx^n|a_i\in\omega,0\leq i\leq n\}$ (1.5)

Addition is defined by $\bar{f}\oplus\bar{g}=(a_0+b_0\ a_1+b_1\ a_2+b_2\ \ldots\ a_n+b_n)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}$ (1.5.1)

Multiplication is defined by $\bar{f}\odot\bar{g}=(a_0\cdot b_0\ a_1\cdot b_1\ a_2\cdot b_2\ \ldots\ a_n\cdot b_n)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}$ (1.5.2)

Observe that $a_0,a_1,a_2,\ldots,a_n,b_0,b_1,b_2,\ldots,b_n\in\omega$ and ω is a field provides $(\omega^{n+1}[x],\oplus)$ is an additive abelian group where $\oplus$ is defined (1.5.1). (1.5.3)

Similarly, $(\omega^{n+1}[x],\odot)$ is a semi group in which $(1\ 00\ldots0)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}=1$ acts as the unity and commutative property and distributive properties hold. (1.5.4)

However, $(0a_i\ 00\ldots0)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}\odot\big(000a_j0\ldots0\big)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}=(00\ 00\ldots0)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}=\bar{0}$ the zero element in (1.5) in view of the definition (1.5.2) (1.5.5)

(Note that in view of (1.5.1) through (1.5.5), it can be stated that (1.5) is a commutative ring with divisors of zero.) (1.6)

Taking the non-negative square root,

$d(\bar{f},\bar{g})=\max\sqrt{\sum_{k=0}^{m}((a_k-b_k)x^k)^2}$ Where$|\bar{x}^{\,T}|\leq1,\bar{x}=(1,x,x^2,\ldots,x^n)\in n+1-cell\subset\mathbb{R}^{n+1}$, it can be seen that $d(\bar{f},\bar{g})\geq0$ (1.6.1)

Observe that $\bar{x}^{\,T}=\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}$ and $|\bar{x}|=\sqrt{\frac{1-x^{2n+1}}{1-x}}$

$d(\bar{f},\bar{g})=0$ if and only if $\sum_{k=0}^{n}\big((a_k-b_k)x^k\big)^2=0$

Since not all x^k equal to 0, sum of the squares is 0 if and only if each term is 0.

If and only $a_k-b_k=0,0\leq k\leq n$

That is, $d(\bar{f},\bar{g})=0$ If and only if $\bar{f}=\bar{g}$. (1.6.2)

$$\sqrt{\sum_{k=0}^{n}((a_k-b_k)x^k)^2}\leq\sqrt{\sum_{k=0}^{n}((a_k-c_k)x^k)^2}+\sqrt{\sum_{k=0}^{n}((c_k-b_k)x^k)^2}$$

Or, $d(\bar{f},\bar{g})\leq d(\bar{f},\bar{h})+d(\bar{h},\bar{g})$ (1.6.3)

$\sqrt{\sum_{k=0}^{n}((a_k-b_k)x^k)^2}=\sqrt{\sum_{k=0}^{n}((b_k-a_k)x^k)^2}$ or $d(\bar{f},\bar{g})=d(\bar{g},\bar{f})$ (1.6.4)

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(1.6.1) through (1.6.4), $(\omega^{n+1}[x], d)$ is a metric space. (1.7)

2. DUAL SPACE OVER THE METRIC SPACE

$\omega^{n+1}[x](\omega)$:

Definition: The definition (1.5.3) can also be considered as the addition of vectors on $\omega^{n+1}[x]$ and defining $\alpha\,\bar{f} = (aa_0\ aa_1\ aa_2 \dots aa_n)\begin{pmatrix}1\\x\\x^2\\\vdots\\x^n\end{pmatrix}$ as the α scalar multiplication of vector $\bar{f}$, $\omega^{n+1}[x](\omega)$ is a linear space of dimension n +1. (2.1)

If $\omega^{n+1}[x](\omega)$ and $\omega^{m+1}[x](\omega)$ are two linear spaces over the same polynomial ring $\omega[x\ ]$ and $\boldsymbol{\lambda}$: $\omega^{n+1}[x](\omega) \to \omega^{m+1}[x](\omega)$is a function satisfying $\boldsymbol{\lambda}(\bar{f}) = \bar{g}$

$\boldsymbol{\lambda}(\bar{f}) = \boldsymbol{\lambda}(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n) = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m$ (2.2)

$\boldsymbol{\lambda}(\alpha\bar{f} \oplus \beta\bar{g}) = \alpha\boldsymbol{\lambda}(\bar{f}) + \beta\boldsymbol{\lambda}(\bar{g})$ will verify that $\boldsymbol{F}$ is a linear transformation. (2.3)

$\omega^{n+1}[x](\omega)$ is a linear space of dimension $n+1$ and so, let the basis be

$S = \{\bar{f}_0, \bar{f}_1, \bar{f}_2, \dots, \bar{f}_n\}$ and that of $\omega^{m+1}[x](\omega)$ be $S_1 = \{\bar{g}_0, \bar{g}_1, \bar{g}_2, \dots, \bar{g}_m\}$ (2.4)

$L(\omega^{n+1}[x](\omega), \omega^{m+1}[x](\omega))$ is the set of linear transformations defined from

$\omega^{n+1}[x](\omega)$ into $\omega^{m+1}[x](\omega)$. It can be seen that $L(\omega^{n+1}[x](\omega), \omega^{m+1}[x](\omega))$ is also a linear space under $\boldsymbol{\lambda}\mp\boldsymbol{\mu}$ as the addition of linear transformations and $\theta\boldsymbol{F}$as the scalar multiplication of linear operators. (2.5)

This linear space is of dimension $(n+1)\times(m+1)$

If $\bar{f}$=$\gamma_0\bar{f_0} + \gamma_1\bar{f_1} + \gamma_2\bar{f_2} + \cdots + \gamma_n\bar{f_n}$

$$= \gamma_0\sum_{i=0}^{n}\alpha_{0i}x^i + \gamma_1\sum_{i=0}^{n}\alpha_{1i}x^i + \cdots + \gamma_n\sum_{i=0}^{n}\alpha_{ni}x^i$$

$= \{\gamma_0\sum_{i=0}^{n}\alpha_{0i}$+$\gamma_1\sum_{i=0}^{n}\alpha_{1i} + \cdots + \gamma_n\sum_{i=0}^{n}\alpha_{ni}\}x^i$, then

$\boldsymbol{\lambda}(\bar{f})$= $\delta_0\{\gamma_0\sum_{i=0}^{n}\alpha_{0i}$+$\gamma_1\sum_{i=0}^{n}\alpha_{1i} + \cdots + \gamma_n\sum_{i=0}^{n}\alpha_{ni}\}x^0 +$

$\delta_1\{\gamma_0\sum_{i=0}^{n}\alpha_{0i}$+$\gamma_1\sum_{i=0}^{n}\alpha_{1i} + \cdots + \gamma_n\sum_{i=0}^{n}\alpha_{ni}\}x^1 + \cdots +$

$\delta_m\{\gamma_0\sum_{i=0}^{n}\alpha_{0i}$+$\gamma_1\sum_{i=0}^{n}\alpha_{1i} + \cdots + \gamma_n\sum_{i=0}^{n}\alpha_{ni}\}x^m$

$$= \begin{bmatrix} \delta_0\gamma_0\sum_{i=0}^{n}\alpha_{0i} & \delta_1\gamma_0\sum_{i=0}^{n}\alpha_{0i} & \delta_2\gamma_0\sum_{i=0}^{n}\alpha_{0i} & \delta_m\gamma_0\sum_{i=0}^{n}\alpha_{0i} \\ \delta_0\gamma_1\sum_{i=0}^{n}\alpha_{1i} & \delta_1\gamma_1\sum_{i=0}^{n}\alpha_{1i} & \delta_2\gamma_1\sum_{i=0}^{n}\alpha_{1i} & \cdots\ \delta_m\gamma_1\sum_{i=0}^{n}\alpha_{1i} \\ \vdots & \vdots & \vdots & \vdots \\ \delta_0\gamma_n\sum_{i=0}^{n}\alpha_{ni} & \delta_1\gamma_n\sum_{i=0}^{n}\alpha_{ni} & \delta_2\gamma_n\sum_{i=0}^{n}\alpha_{ni} & \delta_m\gamma_n\sum_{i=0}^{n}\alpha_{ni} \end{bmatrix}\begin{bmatrix}1\\x\\x^2\\\vdots\\x^m\end{bmatrix}$$

$= M\hat{\boldsymbol{x}}^T$ Where M is the matrix representation of the transformation $\boldsymbol{\lambda}$. (2.6)

This matrix product representation is with respect to the bases S and S_1 relative to transformation $\boldsymbol{\lambda}$. So, this representation can also be denoted by $M = (\boldsymbol{\lambda}, S, S_1)$. (2.7)

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Definition: if $\boldsymbol{\lambda}: \omega^{n+1}[x](\omega) \to \omega^{m+1}[x](\omega)$ is a linear transformation, then the non – negative real number **norm** of $\boldsymbol{\lambda}$is defined by

$\|\boldsymbol{\lambda}\| = \sup\left\{|M_i|\sqrt{\sum_{k=0}^{m} x^k} \,/\, x \in \mathbb{R},\ \sqrt{\sum_{k=0}^{m} x^k} \leq 1\right\}$ and $|M_i|$ is the non-negative determinant of the largest square submatrix of *M*. (2.8)

Chapter 3:

There can be infinitely many such transformations $\boldsymbol{\lambda}: \omega^{n+1}[x](\omega) \to \omega^{m+1}[x](\omega)$

$L\,(\omega^{n+1}[x](\omega), \omega^{m+1}[x](\omega))$ is the collection of all linear transformations defined is also a vector space under $(\xi\,\boldsymbol{\lambda_1} \mp \eta\,\boldsymbol{\lambda_2})(\bar{f}) = \xi\boldsymbol{\lambda_1}(\bar{f}) \mp \eta\boldsymbol{\lambda_2}(\bar{f})$ for any $\xi, \eta \in \omega$ (3.1)

In view of the definitions (1.0), (1.6) and (1.11), if $\boldsymbol{\lambda}_i \in L\,(\omega^{n+1}[x](\omega), \omega^{m+1}[x](\omega))$ and $d_1(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}) < \delta$ a small positive real number, then by the linearity of $\boldsymbol{\lambda}_i$, it can be seen that (1.9) gives $\left\|\boldsymbol{\lambda_1}(\bar{f}) - \boldsymbol{\lambda_1}(\bar{g})\right\|$

$$= \sup\left\{|M_i' - N_i'|\sqrt{\sum_{k=0}^{m} x^k} \,/\, \bar{x} \in \mathbb{R}^{m+1}, \qquad |\bar{x}| = \sqrt{\sum_{k=0}^{m} x^k} \leq 1\right\}$$

$$< \varepsilon$$

Where $|M_i' - N_i'|$ is the non – negative determinant of the largest square submatrix of $(\boldsymbol{\lambda}, S, S_1)$ as shown in (1.13).

This verifies the continuity property of the members of $L\,(\omega^{n+1}[x](\omega), \omega^{m+1}[x](\omega))$

Let the set of all continuous transformation in $L\,(\omega^{n+1}[x](\omega), \omega^{m+1}[x](\omega))$be denoted by $\mathcal{C}(L)$. (3.2)

With reference to the definition 2.8, it follows $(\mathcal{C}(L), \|\ \ \|)$ is a metric space. (3.3)

While an $(n+1) \times (m+1) -$cell is compact, every Cauchy sequence in $\mathcal{C}(L)$ is converging in $\mathcal{C}(L)$ itself showing that $\mathcal{C}(L)$ is a complete metric space. In other words, $\mathcal{C}(L)$ is a Banach space (3.4)

If $\mathfrak{B}$ is a Banach space upon which the ordering relation $\leq$ is defined by $\|\boldsymbol{\lambda}_1\| \leq \|\boldsymbol{\lambda}_2\|$ using $\|\boldsymbol{\lambda}\| = sup\left\{|\boldsymbol{\lambda}(x)|: \boldsymbol{\lambda} \in \mathcal{C}(L), x \in (n+1) \times (m+1) - cell \& |\boldsymbol{\lambda}(x)| \in \mathbb{R}^+ \cup \{0\}\right\}$, then $\|\boldsymbol{\lambda}\| \leq \|\boldsymbol{\lambda}\|$ showing $\leq$ is a symmetric relation. $\|\boldsymbol{\lambda}_1\| \leq \|\boldsymbol{\lambda}_2\|$ and $\|\boldsymbol{\lambda}_2\| \leq \|\boldsymbol{\lambda}_1\|$ need not imply $\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2$. This show $\leq$ is a symmetric relation. (3.5)

Further, $\|\boldsymbol{\lambda}_1\| \leq \|\boldsymbol{\lambda}_2\|$ and $\|\boldsymbol{\lambda}_2\| \leq \|\boldsymbol{\lambda}_3\|$ implies $\|\boldsymbol{\lambda}_1\| \leq \|\boldsymbol{\lambda}_3\|$ showing $\leq$ is a transitive relation. Therefore, $\leq$ is an equivalence relation on. (3.6)

In other words, $(\mathfrak{B}, \leq)$ is not a partially ordered set or *POSET*. (3.7)

Let us consider $\mathfrak{B}^* = \{\boldsymbol{\lambda} \in \mathfrak{B}: \|\boldsymbol{\lambda}_1\| \leq \|\boldsymbol{\lambda}_2\| \& \|\boldsymbol{\lambda}_2\| \leq \|\boldsymbol{\lambda}_1\| \Longrightarrow \boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2\}$. $(\mathfrak{B}^*, \leq)$ is a partially ordered set. (3.8)

Theorem: $\mathfrak{B}^*$ is subalgebra of $\mathfrak{B}$ both in the form of a ring as well as a linear space. It is the smallest σ- algebra containing $\mathfrak{B}$ (3.9)

Chapter 4:

A polynomial of degree $\leq m$ is verified as a continuous function in $\mathfrak{B}^*$.

A set *A* is μ – measurable if there is a function $\mu: \mathfrak{B} \to \mathfrak{N}$ where $\mathfrak{B}$ is the set of subsets of a set *X* and$A \in \mathfrak{B}$, $\mathfrak{N}$ is the set of non-negative real numbers. μ has to satisfy (i) $\mu\square = 0$, (ii) $A \dot{\subseteq} B \implies \mu A \leq \mu B$ and (iii) $\mu\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \mu A_i$ for a countable collection of members A_i of $\mathfrak{B}$. (4.1)

A function $\boldsymbol{\lambda}$ is said to be μ – measurable if $\{x: \boldsymbol{\lambda}(x) \leq \alpha\}$ is measurable for any real number α. (4.2)

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Observe that $\mu A \leq \mu B$ for every pair of members $A, B \in \mathfrak{B}$ with $A \subseteq B$ which is the ordering relation on $\mathfrak{B}$. Also certain members of $\mathfrak{B}$ are not comparable and so, the formed set is a poset.

The definition of $\|\lambda\|$ is assigned to the absolute value of the determinant of the representative matrix, the definition (4.1) is suitable in $\mathfrak{B}^*$.

If $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2$ are μ – measurable, then $\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2$ is μ – measurable.

If a is a real number and $\boldsymbol{\lambda}$ is μ – measurable, then $a\boldsymbol{\lambda}$ is μ – measurable.

(3.1) and (3.2) allows the set of all measurable functions to form a linear space shown in (2.2) through (2.6).

This may be denoted by $\mathcal{C}(L, \mathfrak{B}, \mu)$ (4.3)

Choose two non-negative real numbers $p\&q$ such that $\frac{1}{p} + \frac{1}{q} = 1$ and

$L^1 = \{\boldsymbol{\lambda} \in \mathcal{C}(L, \mathfrak{B}, \mu) : |M| < \infty\}$ (4.4)

Observe that for every $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2 \in L^1$, $|\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2| = |M_1 + M_2| \leq |\boldsymbol{\lambda}_1| + |\boldsymbol{\lambda}_2| < \infty$ and

$|\alpha \boldsymbol{\lambda}_1| = |\alpha||M| < \infty$ Shows that L^1 is also a linear space. (4.5)

Define 'p^{th} – norm' a non-negative real number on L^1 by $\|\lambda\|_p = |M|$ satisfies the definition 2.8.

Define a relation '$\leq$' on L^1 by (2.8) to show $\|\boldsymbol{\lambda}_1\|_1 \leq \|\boldsymbol{\lambda}_2\|_1$ for every $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2 \in L^1$ (4.6)

$(L^1, \leq)$ is a *POSET* and when $\frac{1}{p} + \frac{1}{q} = 1$ is satisfied. (4.7)

Define $\vee$ on $\mathfrak{B}^* = L^1 =$ by $\boldsymbol{\lambda}_1 \vee \boldsymbol{\lambda}_2 = \|\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2\|_1 = \rho(M_1 + M_2)$ (4.8)

and $\wedge$ by $\boldsymbol{\lambda}_1 \wedge \boldsymbol{\lambda}_2 = \|\boldsymbol{\lambda}_1 \boldsymbol{\lambda}_2\|_1$ (4.9)

Clearly, $|M_1| < \infty, |M_2| < \infty$ says $|M_1 + M_2| < \infty$, $|M_1 M_2| < \infty$ showing

$\boldsymbol{\lambda}_1 \vee \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_1 \wedge \boldsymbol{\lambda}_2 \in L^1$ and the definition of $\mathfrak{B}^*$ will confirm the uniqueness of $\vee$ and $\wedge$ in

Also, $\boldsymbol{\lambda}_1 \vee \boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_2 \vee \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_1 \wedge \boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_2 \wedge \boldsymbol{\lambda}_1$ and other properties of Lattice are satisfied to show $(L^1, \leq)$ is a lattice. (4.10)

Further, by properties of matrices, it can be seen that $\|\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2\|_1 \leq \|\boldsymbol{\lambda}_1\|_1 + \|\boldsymbol{\lambda}_2\|_1$ (4.11)

Also, $\|\boldsymbol{\lambda}_1 \boldsymbol{\lambda}_2\|_1 \leq \|\boldsymbol{\lambda}_1\|_1 \|\boldsymbol{\lambda}_2\|_1$ (4.12)

Now consider $(\boldsymbol{\lambda}_1 \vee \boldsymbol{\lambda}_2) \wedge \boldsymbol{\lambda}_3$

$= (\|\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2\|_1)\|\boldsymbol{\lambda}_3\|_q$ by (4.8) &(4.9)

$\leq \left(\|\boldsymbol{\lambda}_1\|_p + \|\boldsymbol{\lambda}_2\|_p\right)\|\boldsymbol{\lambda}_3\|_q$ by (3.6)

$\leq \|\boldsymbol{\lambda}_1\|_p \|\boldsymbol{\lambda}_3\|_q + \|\boldsymbol{\lambda}_2\|_p \|\boldsymbol{\lambda}_3\|_q$ While p^{th} norm is a real number

$\leq (\boldsymbol{\lambda}_1 \wedge \boldsymbol{\lambda}_3) \vee (\boldsymbol{\lambda}_2 \wedge \boldsymbol{\lambda}_3)$ (4.13)

In view of (4.10) and (4.13), $(L^1, \leq)$ is a not a distributive lattice respect to the definitions (4.8) & (4.9)

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