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FUZZY LATTICE K- OPERATOR SOFT GROUP

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Abstract
In this paper we defined a new algebraic structure which consists of group, lattice and a fuzzy set. In particular we studied here the effect of a non empty set called as operator set on fuzzy soft groups.

Keywords: Lattice K- operator group , Lattice K- operator soft group, Fuzzy lattice K- operator soft group.

1.Introduction

Mathematics is a branch of science where the results are certain and logically proved. But most of the things in this world are uncertain. There are theories like probability which is used to measure the possibility of happenings of events. On similar line yes-no concepts are further addressed with the help of grades and a fuzzy set concept was introduced by Zadeh[11].Further Molodsov [5] started a theory of soft sets which is a model to study vagueness and uncertainty.Molodsov worked on various applications of soft sets which help in decision making in uncertain cases.Maji, Biswas and Roy [3, 12, 13, 14] has done a rigorous work in applications of soft sets. Akta³ and Çagman [9] generalized the concept of soft sets and introduced the concept of soft group. In 1971 Rosenfeld[2] added a new algebraic structure 'fuzzy group' in the theory of fuzzy sets and groups.Abdukkadir,Aygunoglu, Halis Aygun [1] introduced the concept of fuzzy soft groups. In this paper we applied the concept of operator group introduced by Subramanian, Nagrajan, Chellappa [15] to fuzzy soft groups. Here K is treated as aoperator set. We defined fuzzy lattice K- operator soft group and studiesits various properties.

2. Preliminaries

Definition 2.1. [9]Let X be a group and (λ ; A) be a soft set over X. Then (λ ; A) is said to be a soft group over X iff λ (a) is a subgroup of X for all a ϵ A.
Definition 2.2.[2] Let λ: X → [0, 1] is a fuzzy set & G is a subset of X. λ is a fuzzy group if it satisfy following two conditions
i) $\lambda\ (xy) \geq$ mini $\{\ \lambda(x),\ \lambda\ (y)\}$
ii) $\lambda\ (x^{-1}) \geq \lambda(x)$ where x, yϵ G
Definition 2.3.[1]Let X be a group and (λ ; A) be a soft group over X. Then (λ; A) is said to be a fuzzy soft group over X iff for each a ϵ A and x; y ϵ X,
(1) $\lambda a(x\ y) \geq$ mini$(\lambda a(x),\lambda a(y))$
(2) $\lambda a(x^{-1}) \geq \lambda a(x)$
Definition 2.4.[7] Lattice group is an algebraic structure (G, . , R) if it satisfy two conditions 1) G is a group w.r.t '.' 2) G is a lattice w.r.t R
Definition 2.5.[15] A group G is said to be a K- operator group if k x ϵ G where k ϵ K (any non empty set called as Operator set) and for all xϵ G.
Definition 2.6. A lattice group G is said to be a lattice K- operator group if k x ϵ G where k ϵ K (any non empty set called as Operator set) and for all xϵ G.
Definition 2.7. Let X be a lattice group and (λ ; A) be a soft group over X. Then (λ ; A) is said to be a fuzzy soft lattice group over X iff for each a ϵ A and x; y ϵ X,
(1) $\lambda a(x\ y) \geq$ mini$(\lambda a(x),\lambda a(y))$

(2) $\lambda a(x^{-1}) \geq \lambda a(x)$
(3) $\lambda a(x \vee y) \geq \text{mini}(\lambda a.(x), \lambda a(y))$
(4) $\lambda a(x \wedge y) \geq \text{mini}(\lambda a.(x), \lambda a(y))$
for each $a \in A$ and $x; y \in X$,

Definition 2.8. Let X be a lattice K- operator group and (λ ; A) be a soft group over X. Then (λ ; A) is said to be a fuzzy lattice K-operator soft group(FL K-operator soft group) over X iff for each $a \in A$ and $x; y \in X$,
(1) $\lambda a(kx\, ky) \geq \text{mini}(\lambda a(k x), \lambda a(k y))$
(2) $\lambda a (kx^{-1}) \geq \lambda a(k x)$
(3) $\lambda a(kx \vee k y) \geq \text{mini}(\lambda a(k x), \lambda a(k y))$
(4) $\lambda a(k x \wedge k y) \geq \text{mini}(\lambda a(k x), \lambda a(k y))$
for each $a \in A$ and $x; y \in X$, $k \in K$

Definition 2.9. **[14]** Let (λ ,A) and (δ, B) are two fuzzy soft sets over X_1 and X_2 respectively then their product (λ x δ, Ax B) is defined by $(\lambda \times \delta)_{(a, b)} (x_1, x_2) = \lambda_a (x_1) \wedge \delta_b (x_2)$

Definition 2.10.**[14]**Intersection of two fuzzy soft sets (λ; A) and (δ ; B) over a common universe X is the fuzzy soft set (h ; C) where $C = A \cap B$ and $h_c = \lambda_c \wedge \delta_c$ for all $c \in C$

Definition 2.11. [14] Union of two fuzzy soft sets (λ; A) and (δ ; B) over a common universe X is the fuzzy soft set (h ; C) where $C = A \cup B$ and

$$h_c = \begin{cases} \lambda_c & \text{if } c \in A - B \\ \delta_c & \text{if } c \in B - A \\ \lambda_c \vee \delta_c & \text{if } c \in A \cap B \end{cases} \qquad \text{for all } c \in C$$

Definition 2.12.The fuzzy K- operator soft group (λ, A) over X is called an abelian fuzzy K-operator soft group if for each, $a \in A$ $\lambda_a (k\, y\, k\, x) = \lambda_a (k\, x\, k\, y)$, for all $x,y \in X$

Definition 2.13. Let X be a group, (λ, A) and (δ, A) are two fuzzy K-operator soft groups over X then (λ, A) is said to be conjugate to (δ, A) if there exist $x \in X$ such that for all $y \in X$,
$\lambda_a (k\, y) = \delta_a(k\, x^{-1}\, yx)$, $a \in A$

3. PROPERTIES OF FUZZY LATTICE K- OPERATOR SOFT GROUP

Proposition 3.1. If (λ ; A) be a FL K- operator soft group. e is the unit element of X.
Then i) $\lambda a (k\, x^{-1}) = \lambda a (k\, x)$
ii) if $\lambda a(k\, x\, y) = \lambda a(k\, x\, k\, y)$ then $\lambda a (k\, e) \geq \lambda a (k\, x)$

Proof- Consider (λ ; A) is a FL K-operator soft group.
i) We have $\lambda a((k\, x^{-1}) \geq \lambda a(k\, x)$
$\lambda a((k\, x) = \lambda a((k\, (x^{-1})^{-1}) \geq \lambda a(k\, x^{-1})$
Therefore $\lambda a((k\, x) \geq \lambda a(k\, x^{-1})$. Hence $\lambda a (k\, x^{-1}) = \lambda a (k\, x)$
ii) $\lambda a (k\, e) = \lambda a((k\, x\, x^{-1}) = \lambda a((k\, x\, k\, x^{-1}) \geq \text{mini} \{\lambda a(k\, x), \lambda a(k\, x^{-1})\}$
$= \text{mini} \{\lambda a(k\, x), \lambda a(k\, x)\} = \lambda a (k\, x)$

Proposition 3.2. If (λ; A) be a lattice K- operator soft group.
Then (λ ; A) is a FL K-operatorsoft group iff for each $a \in A$ and $x, y \in X$, $\lambda a((k\, x\, y^{-1}) \geq \text{mini} \{\lambda a(k\, x), \lambda a(k\, y)\}$ and $\lambda a(k\, x\, y) = \lambda a(k\, x\, k\, y)$

Proof- Consider (λ ; A) is a FL K-operator soft group.
$\lambda a((k\, x\, y^{-1}) = \lambda a((k\, (y\, x^{-1})^{-1}) \geq \lambda a (k\, y\, x^{-1}) = \lambda a (k\, y\, k\, x^{-1}) \geq \text{mini} \{\lambda a(k\, y), \lambda a(k\, x^{-1})\}$
$\geq \text{mini} \{\lambda a(k\, y), \lambda a(k\, x)\} = \text{mini} \{\lambda a(k\, x), \lambda a(k\, y)\}$
Conversely consider $\lambda a((k\, x\, y^{-1}) \geq \text{mini} \{\lambda a(k\, x), \lambda a(k\, y)\}$
$\lambda a (ke) = \lambda a((k\, x\, x^{-1}) \geq \text{mini} \{\lambda a(k\, x), \lambda a(k\, x)\} = \lambda a (k\, x)$
$\lambda a((k\, x^{-1}) = \lambda a((k\, e\, x^{-1}) \geq \text{mini} \{\lambda a(k\, e), \lambda a(k\, x)\} = \lambda a (k\, x)$
$\lambda a((k\, x\, k\, y) = \lambda a((k\, x\, y) = \lambda a((k\, x\, (y^{-1})^{-1}) \geq \text{mini} \{\lambda a(k\, x), \lambda a(k\, y^{-1})\}$
$\geq \text{mini} \{\lambda a(k\, x), \lambda a(k\, y)\}$

Proposition 3.3. If (λ; A) and(δ ; B) be two FL K- operator soft groups over X. Then the intersection $(\lambda; A) \cap (\delta; B)$ is a FL K- operator soft group over X.

Proof- Let $(\lambda; A) \cap (\delta; B) = (h; C)$ where $C = A \cap B$ and $h_c = \lambda_c \wedge \delta_c$ where

$h_c(x) = \lambda_c(x) \wedge \delta_c(x)$ for all $c \in C$ and $x \in X$.

$\lambda_c(k\,x\,k\,y) \geq \text{mini}(\lambda_c(k\,x), \lambda_c(k\,y)), \delta_c(k\,x\,k\,y) \geq \text{mini}(\delta_c(k\,x), \delta_c(k\,y))$

$\lambda_c(k\,x^{-1}) \geq \lambda_c(k\,x)$, $\delta_c(k\,x^{-1}) \geq \delta_c(k\,x)$

$\lambda_c(k\,x \vee k\,y) \geq \text{mini}(\lambda_c(k\,x), \lambda_c(k\,y))$, $\delta_c(k\,x \vee k\,y) \geq \text{mini}(\delta_c(k\,x), \delta_c(k\,y))$

$\lambda_c(k\,x \wedge k\,y) \geq \text{mini}(\lambda_c(k\,x), \lambda_c(k\,y))$, $\delta_c(k\,x \wedge k\,y) \geq \text{mini}(\delta_c(k\,x), \delta_c(k\,y))$

i) $h_c(k\,x\ k\,y) = \lambda_c \wedge \delta_c(k\,x\,k\,y)$
$= \lambda_c(k\,x\ k\,y) \wedge \delta_c(k\,x\,k\,y)$
$\geq \text{mini}(\lambda_c(k\,x), \lambda_c(k\,y)) \wedge \text{mini}(\delta_c(k\,x), \delta_c(k\,y))$
$= \text{mini}(\lambda_c(k\,x) \wedge \delta_c(k\,x), \lambda_c(k\,y) \wedge \delta_c(k\,y))$
$= \text{mini}(\lambda_c \wedge \delta_c(k\,x), \lambda_c \wedge \delta_c(k\,y))$
$= \text{mini}(h_c(k\,x), h_c(k\,y))$

ii) $h_c(k\,x^{-1}) = h_c(k\,x^{-1}) = \lambda_c \wedge \delta_c(k\,x^{-1}) = \lambda_c(k\,x^{-1}) \wedge \delta_c(k\,x^{-1})$
$= \lambda_c(k\,x) \wedge \delta_c(k\,x) = \lambda_c \wedge \delta_c(k\,x) = h_c(k\,x)$

iii) $h_c(k\,x \vee k\,y) = \lambda_c \wedge \delta_c(k\,x \vee k\,y)$
$= \lambda_c(k\,x \vee k\,y) \wedge \delta_c(k\,x \vee k\,y)$
$\geq \text{mini}(\lambda_c(k\,x), \lambda_c(k\,y)) \wedge \text{mini}(\delta_c(k\,x), \delta_c(k\,y))$
$= \text{mini}(\lambda_c(k\,x) \wedge \delta_c(k\,x), \lambda_c(k\,y) \wedge \delta_c(k\,y))$
$= \text{mini}(\lambda_c \wedge \delta_c(k\,x), \lambda_c \wedge \delta_c(k\,y))$
$= \text{mini}(h_c(k\,x), h_c(k\,y))$

iv) $h_c(k\,x \wedge k\,y) = \lambda_c \wedge \delta_c(k\,x \wedge k\,y)$
$= \lambda_c(k\,x \wedge k\,y) \wedge \delta_c(k\,x \wedge k\,y)$
$\geq \text{mini}(\lambda_c(k\,x), \lambda_c(k\,y)) \wedge \text{mini}(\delta_c(k\,x), \delta_c(k\,y))$
$= \text{mini}(\lambda_c(k\,x) \wedge \delta_c(k\,x), \lambda_c(k\,y) \wedge \delta_c(k\,y))$
$= \text{mini}(\lambda_c \wedge \delta_c(k\,x), \lambda_c \wedge \delta_c(k\,y))$
$= \text{mini}(h_c(k\,x), h_c(k\,y))$

Therefore $(\lambda; A) \cap (\delta; B)$ is a FL K- operator soft group over X.

Proposition 3.4 If $(\lambda; A)$ and $(\delta; B)$ be two FL K- operator soft groups over X. Then the union $(\lambda; A) \cup (\delta; B)$ is a FL K- operator soft group over X if $A \cap B = \emptyset$.

Proof - Let $(\lambda; A) \cap (\delta; B) = (h; C)$ where $C = A \cup B$.

As $A \cap B = \emptyset$, $c \in A\text{-}B$ or $c \in B\text{-}A$.

If $c \in A\text{-}B$ then $h_c = \lambda_c$ which is a FL K-operator soft group over X and If $c \in B\text{-}A$ then $h_c = \delta_c$ which is also a FL K-operator soft group over X.

Proposition 3.5. If $(\lambda; A)$ and $(\delta; B)$ be two FL K- operator soft group over X_1 and X_2 respectively. Then the product $(\lambda \times \delta, A \times B)$ is a FL K- operator soft group over $X_1 \times X_2$.

Proof- i) $(\lambda x\,\delta)_{(a,b)}\, k\,(x_1,x_2)\, k\,(y_1,y_2) = (\lambda x\,\delta)_{(a,b)}\,(k\,x_1 k\,y_1, k\,x_2\,k y_2)$
$= \lambda_a(k\,x_1 k\,y_1) \wedge \delta_b(k\,x_2 k y_2)$
$\geq \text{mini}\{\lambda_a(k\,x_1), \lambda_a(k y_1)\} \wedge \text{mini}\{\delta_b(k\,x_2), \delta_b(k y_2)\}$
$= \text{mini}\{\lambda_a(k\,x_1) \wedge \delta_b(k\,x_2), \lambda_a(k y_1)\} \wedge \delta_b(k y_2)\}$
$= \text{mini}\{(\lambda x\,\delta)_{(a,b)}\, k\,(x_1, x_2), (\lambda x\,\delta)_{(a,b)}\, k\,(y_1, y_2)\}$

ii) $(\lambda x\,\delta)_{(a,b)}\, k\,(x_1,x_2)^{-1} = (\lambda x\,\delta)_{(a,b)}\, k\,(x_1^{-1}, x_2^{-1})$
$= (\lambda x\,\delta)_{(a,b)}\,(k\,x_1^{-1}, k\,x_2^{-1})$
$= \lambda_a(k\,x_1^{-1}) \wedge \delta_b(k\,x_2^{-1})$
$\geq \lambda_a(k\,x_1) \wedge \delta_b(k\,x_2)$
$= (\lambda x\,\delta)_{(a,b)}\,(k\,x_1, k\,x_2)$
$= (\lambda x\,\delta)_{(a,b)}\, k\,(x_1,x_2)$

iii) $(\lambda x\,\delta)_{(a,b)}\, k\,(x_1,x_2) \vee k\,(y_1,y_2) = (\lambda x\,\delta)_{(a,b)}\,(k\,x_1 \vee k\,y_1, k\,x_2 \vee k\,y_2)$
$= \lambda_a(k\,x_1 \vee k\,y_1) \wedge \delta_b(k\,x_2 \vee k y_2)$
$\geq \text{mini}\{\lambda_a(k\,x_1), \lambda_a(k y_1)\} \wedge \text{mini}\{\delta_b(k\,x_2), \delta_b(k y_2)\}$
$= \text{mini}\{\lambda_a(k\,x_1) \wedge \delta_b(k\,x_2), \lambda_a(k y_1)\} \wedge \delta_b(k y_2)\}$
$= \text{mini}\{(\lambda x\,\delta)_{(a,b)}\, k\,(x_1, x_2), (\lambda x\,\delta)_{(a,b)}\, k\,(y_1, y_2)\}$

iv) $(\lambda x\,\delta)_{(a,b)}\, k\,(x_1,x_2) \wedge k\,(y_1,y_2) = (\lambda x\,\delta)_{(a,b)}\,(k\,x_1 \wedge k\,y_1, k\,x_2 \wedge k\,y_2)$
$= \lambda_a(k\,x_1 \wedge k\,y_1) \wedge \delta_b(k\,x_2 \wedge k y_2)$

$\geq$ mini{ λ_a (k x_1) , λ_a (ky_1)} $\wedge$ mini { δ_b (k x_2) , δ_b (ky_2)}
= mini{ λ_a (k x_1) $\wedge$ δ_b (k x_2) , λ_a (ky_1)} $\wedge$ δ_b (ky_2)}
= mini{ $(\lambda x \delta)_{(a,b)}$ k (x_1, x_2) , $(\lambda x \delta)_{(a,b)}$ k (y_1, y_2)}

Proposition 3.6. If (λ; A) and (δ ; B) be two FL K- operator soft abelian groups over X_1 and X_2 respectively. Then the product (λx δ, Ax B) is a FL K- operator soft abelian groups over X_1 x X_2.

Proof - $(\lambda x \delta)_{(a, b)}$ k (x_1,x_2) k (y_1, y_2) = $(\lambda x \delta)_{(a, b)}$ (k x_1k y_1, k x_2 k y_2)
=λ_a (k x_1k y_1) $\wedge$ δ_b(kx_2 k y_2)
= λ_a (k y_1k x_1) $\wedge$ δ_b(k y_2 k x_2)
= $(\lambda x \delta)_{(a, b)}$ (k y_1kx_1, k y_2 k x_2)
= $(\lambda x \delta)_{(a, b)}$ k (y_1,y_2) k (x_1, x_2)

Proposition 3.7. If (λ_1; A_1) is conjugate FL K- operator soft group to (δ_1; A_1) and (λ_2; A_2) is conjugate FL K- operator soft group to (δ_2; A_2) then (λ_1 x λ_2; A_1 x A_2) is conjugate FL K- operator soft group to (δ_1x δ_2; A_1 x A_2)

Proof- (λ_1; A_1) is conjugate FSL K- operator group to (δ_1; A_1). Therefore there exist x ϵ X such that $(\lambda_1)_{a1}$ (k y) = $(\delta_1)_{a1}$ (k x^{-1} y x) for all y ϵ X.
(λ_2; A_2) is conjugate FSL K- operator group to (δ_2; A_2). Therefore there exist x ϵ X such that $(\lambda_2)_{a2}$ (k y) = $(\delta_2)_{a2}$ (k x^{-1} y x) for all y ϵ X.

$(\lambda_1 x \lambda_2)_{(a1,a2)}$ (k (y_1, y_2)) = $(\lambda_1)_{a1}$ (k y_1) $\wedge$$(\lambda_1)_{a2}$ (k y_2)
=$(\delta_1)_{a1}$ (k x_1^{-1} y_1 x_1) $\wedge$ $(\delta_2)_{a2}$ (k x_2^{-1} y_2 x_2)
=$(\delta_1 x \delta_2)_{(a1,a2)}$ (k x_1^{-1} y_1 x_1 , k x_2^{-1} y_2 x_2))
=$(\delta_1 x \delta_2)_{(a1,a2)}$ k (x_1^{-1} y_1 x_1 , x_2^{-1} y_2 x_2))

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