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VENDOR MANAGED INVENTORY WITH CONSIDERATION OF CONSIGNMENT

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ABSTRACT

Holding vendor/manufacturer stock in the godown of the buyer is a modern business technique called as vendor/manufacturer-managed stock (VMI) with a stock contract which is consignment nature. The benefit to the consumer is taken into account here as is the detection and removal of faulty products from consignment stock. The manufacturer or vendor sends each manufactured group in a already determined number of groups of products the buyer's godown.. According to market demands, the buyer pulls out these items and inspects them for quality. The portion of storage costs which is not financial in nature, which is taken into account in this numerical model, has an impact on supply chain cost.

Key words: inventory, vendor,supply chain defective items

1. INTRODUCTION

Inventory managementdeals with tracking of movement of goods from manufacturer to consumer with a view to optimize cost incurred. Hence it needs proper recording of usage as well as returned items, purchasing inventory, storing inventory, and profit on selling the inventory are some of the terms. Vendor/manufacturermanaged inventory(VMI) is a strategy to keep the inventory at consigner or customer godowns without loosing ownership

In line with the agreed-upon (VMI policy) inventory level for the particular product, the vendor or manufacturer replaces the stock on behalf of the customer when stock markstouch a specified reorder point.The manufacturer/vendor and the purchaser enter into a legal agreement known as the Consignment Stock (CS) policy. Since there are numerous additional costs before the buyer or customer receives the goods, such as transportation, spillage, etc., the price has escalated by the time it reaches the client. When a manufacturer or seller sells the same items

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online in an effort to help the customer by lowering the unfavourable pricing, the buyer's business suffers.Excessive lead times may hurt online business. Estimating product demand is extremely difficult in both online and offline enterprises.

If the source is located far away, the disposal plans become much more important. The comparison and sensitivity analysis of the financial and nonfinancial holding cost components are the paper's main contributions. To find the numerical answer and maximise overall profit, two approaches are suggested. To test the model, examples, sensitivity analysis including management perceptions are provided.

In Section 2, a summary of relevant literature is provided. The mathematical formulas are provided in Section 3. In Section 4, the findings are examined and numerical examples are given. Section 5 offers a conclusion and summary.

2. LITERATURE SURVEY

Dual-channel supply chain:

A production inventory model which is dual-channel type which includes price components was created by Liu et al. [22]. A unique strategy for return maximisation using dual supply chain channels was established by Batarfi et al. [6]. Free-riding services were included in the two-channel supply chain by Zhou et al. [47]. Further applying the learning and forgetting principle was done by Batarfi et al. [7]. Li et al. [23] looked at specifics of both online and offline company while making inventory selections. Pi et al.'s [28] investigation looked at how continuous demand affected the supply chain, or SC for short .A supply chain model which is fully coordinated with Consignment Stock agreement was described by Chen and Su [10]. Zhu et al developed a model which can meet the requirements for addressing erratic demand. Ying et al. gathered the information from online customers for supply chain which is having dual channel and used it for analysis.

A manageable lead time:

A model of inventory with lead times which are variable in nature and service-level restrictions was given by Jha and Shanker [16]. Yi and Sarker's [42] model of CS policy with controlled lead time is examined. Mandal and Giri underline the value of minimising lead time in the SC in their [24] example of a Supply Chain model which is integrated in nature with a lead time which is able to suit a particular task and quality improvement.

Price-sensitive demand:

A flawed supply chain model with price options for returns of damaged goods was created by Taleizadeh et al. [38]. Taleizadeh et al.'s examination of order placement and pricing on two competing supply chains. Alfares and Ghaithan [1] projected that demand would be based on price. Complementary items have been examined by Zhao et al. [46]. Supply chain with unpredictable demand is addressed by Gupta et al. [14]. A vmi model with price-dependent demand has been discussed by Bieniek [8]. A model for integrated inventories has been described by Dey et al.

CS policy in fuzzy environment:

Demand for items is unpredictable in inventory models; the best method for handling this uncertainty may be fuzzy-based strategies. Björk [3] An eoq model was created by Sadeghi and Niaki [30] using cost as a triangular fuzzy number. A model for an inventory under a trapezoidal fuzzy number has been developed by Kazemi et al. [17]. According to Maity et al. [26], demand rate is hazy or ambiguous. Karthick and Uthayakumar [19] took fuzzy demand into account for implementing VMI-CS policy.

Consignment stock and vendor/manufacturer regulated inventory guidelines:

The inventory model which is an integrated type and Consignment Stock contract between a single producer and customers are both examined by Valentini and Zavanella [139] and Zavanella and Zanoni [45].Battini et al. analysis's of the Consignment Stock agreement policy is also included in [4]. Using a genetic algorithm, the supply chain model has been proposed by Srinivas and Rao [29]. A Supply Chain model for the VMI-CS policy was presented by Ben-Daya et al. [5] and includes a single producer and a number of buyers..Combining VMI and CS agreements will reduce unnecessary spending and let clients add products on schedule. A shared economic lot size for ainventory modelthat includes a manufacturer includinga purchaser was addressed by Goyal [56]. Banerjee [57] assumed that before the shipping, the entire manufacturing batch will be ready. A scenario of several equal-sized shipments of the production lots was put out by Goyal 58[]. This method was improved by Lu [59] so that shipping could occur while manufacturing was in progress. Hill [61] broadened the strategy. An ideal inventory strategy for the advance payment has been devised by Zhang et al. [62]. Economic production quantity (EPQ) models were created by Zhou et al. [63] to assess the best make-or-buy choices. In a vendor/manufacturer-purchaser supply chain, Shu et al. [64] presumed that shipping lag time is exponentially distributed. According to Lee and Kim [66], deterioration is adistinctive propertyof items in a SC. According to Salameh and Jaber [67], nonconforming goods are taken out of the inventory cycle and sold in a secondary market for less money than they were originally purchased for. Consignment stock methods are employed in combined -profit maximizing methods that employ (s, S) policies, according to Valentini and Zavanella [68]. It has been added to the contribution of Braglia and Zavanella [52] to evaluate a variety of circumstances pertaining to the consignment stock policy. Braglia et al. [74] built on the contribution of Braglia and Zavanella [3] by considering a batch processing which is irreversible in nature at the manufacturer's premises. Bazan et al. [75] removed defective items from the manufacturer's facility and discarded or fixed them before sending them to the client in an effort to advance the Braglia and Zavanella [3] approach. A manufacturing and waste get ridden model for a manufacturer-purchaser supply chain with a policy was created by Jaber et al. [76]. A overview of several consignment stock models is provided in [Sarker 81].

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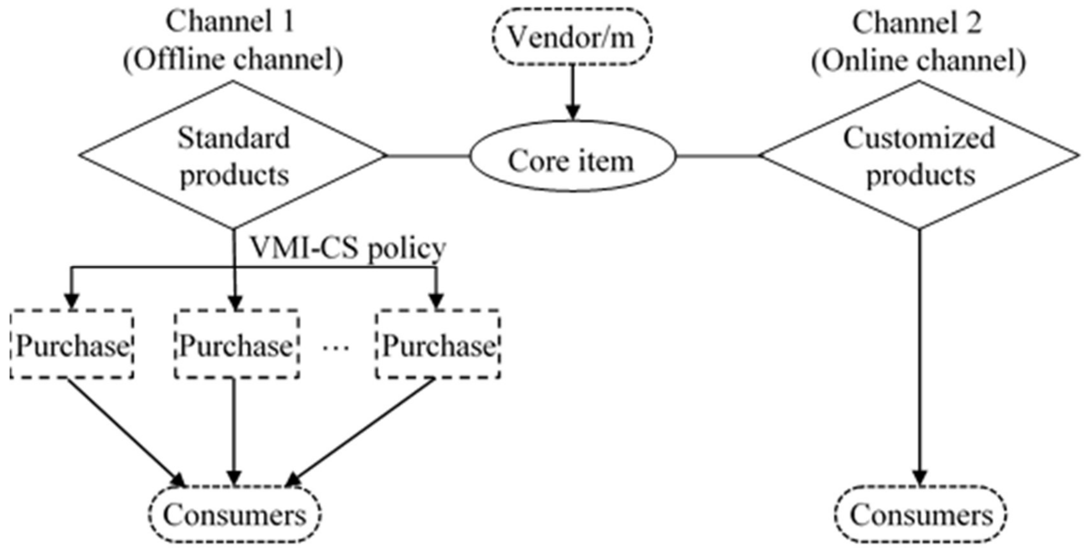

The literature gap in previous research addressed here is:

i) Items must first be inspected at the purchaser's facility to identify the faulty portion of each lot, and then the monetary and nonmonetary holding expenses must be taken into account.

ii) Sensitivity analysis indicating that the demand rate for both standard and customized items is unclear since the supply of goods is split between a single vendor/manufacturer and several customers, taken into account the lead time as anadjustable. The article's goal is to use the entire annual expenditure of the supply chain to estimate the best inventory strategy and the number of lots each batch. Using Figure 2, you can quickly and easily comprehend this concept. The

3. MODEL AND ASSUMPTIONS

The following presumptions are taken into account

1. The relationship between a single supplier or producer and several clients who purchase common commodities is described. The anticipated timeframe is undefined.
2. Every product must have a stringent bigger production rate than demand rate. (i.e., pri>drj, pic > dc) since it is assumed that both standard and customised product production rates are limited.
3. Regardless of the order's production, the setup costs for the manufacturer and the vendor are fixed, continuous costs. Vendor/inventory manufacturer's carrying costs are broken down into two categories: financial and physical. is, and the unit holding expense for a jit buyer during travel is
4. For the purchaser, the lead time llj is comprised of lnj separate parts. The kth portion has a minimal length of mj,k, a normal duration of nj,k, and the cost of crashing per unit of time of eej,k. It also assumes that e,1 lej,2... lej,nij. Starting with the smallest element of ei, each lead time component must be crashed one at a time.

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5. The formula for and is provided by and the cost of crashing the cycle lead time is given by and, if and are the lengths of the lead time components 1, 2, 3,..., and f crashed to their smallest durations. [lj,f, lj,f-1] I lj

The model will be developed using the notations shown below.

Parameters	
di_{rj}	Demand rate (in units/year) for the jth purchaser utilising the offline channel, where j = 1, 2,3,4,,5..., y
di_c	demand requirementin on line channelunits per annum
a_{di}	Key demand, over 0 units per annum
θ_s, $(1–\theta i_s)$	Proportion of each direction's demand that is being met (%)
αi_{rj}	The standard good's price elasticity for buyer j is unit2/rs/year.
αi_{ci}	I = 1, 2,..., z (unit2/rs/year) is the customized item's coefficient of price elasticity.
ρi	Multiple sensitivities
β_{rj}	Sensitivity of the demand dirj's delivery lead time (customers/day)
βi_c	Sensitivity of the demand curve's delivery lead time (customers/day)
ψi_{ci}	Personalized item I = 1, 2, etc., used up 100% of the supply of the core item (%)
pi_r, ip_c	nd<pc< ad<Pr(units/anum) is the manufacturing rate for the basic item that may subsequently be customized.
pn_{cr}, pi_{cci}	Production costs for both standard and custom goods (in rupees per year)
wn_{rj}	Cost of the common item at wholesale to the jth buyer (in rupees per unit)
nS_{mr}, nS_{mc}	Standard and core item setup fees, in relative amounts (rs/setup)
On_{rj}	Ordering fee for the jth purchaser's regular item (rs/ordder)
h_m^{pf}	(Physical and monetary holding costs for the seller or maker are Rs/unit/Year.
i	The maker or seller will pay a financial holding fee of Rs./unit/year for each of the regular item at the purchaser's level..
$i\,h_{rj}^{p}$	(Rs/unit/annum Physical holding costs for the jth buyer

$i\,h_{dj}^{p}$	holding expenses for jth buyers (rs/unit/year)
ti_j	JTH's transportation costs are split into two categories: fixed (rs/shipment) and variable (rs/unit).
υ_j	Cycle length (year)
T_r	For buyer j, the cost of lead time crashing is S/year.
$Bi(l_j)$	Demand rate for the jth buyer using the offline channel, where j = 1, 2,3,4,5,6,7..., y (units/annum)

Decision variables

ni_j	For buyer j, the cost of lead time crashing is S/year.
s_{rij}	Selling cost of a common good for jth buyer (in rupees per unit)
si_{ci}	The cost to sell one customised item (in rupees per unit)
qi_{rj}	Jth buyer's usual item shipping size (number of units per shipment)
iq_c	Production volume of the main product for potential modification (units)
l_j	Lead time duration of jth purchaser (year) (year)

i is financial holding fee

D is the purchaser's rateOfdemand

P is the producers's or supplier's production rate.

Q is the number of goods shipped each lot.

= Amount of loads or transport activities in a batch

Size of the vendor's or manufacturer's batch in Q (Q = q)

Av is the setup fee for a batch from the vendor or manufacturer.

Ab stands for the cost of the buyer's purchase for everylot

= Percentage of faulty products in a lot

X is the rate of buyer screening.

Cost of unit inspectionfor the buyer, d.

HB stands for purchasers Unit Holding Cost (monetary and nonmonetary expenses)

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Hb is the purchaser's unit holding expense (only monetaryportion)

Hb is the purchaser's unit holding expense (non monetary portion)

hv = The manufacturer's unit holding expense (monetary and nonmonetary expenses)

hv = The unit holding cost of the producer or supplier (only monetary portion)

Hb is the unit holding cost for manufacturer (non nonmonetary portion)

Cycle length is denoted by T. Each batch is divided into many lots of comparable sizes and sent to the customer. The producer or seller stores goods in the buyer's godown in compliance with the consignment stock regulations. According to market demand, the buyer sells this shares. It is presumed that in every lot that the customer withdraws has a certain portion of damaged goods. Products that are damaged in the lot are removed. Each lot of these defective goods is discarded. Figure 1 depicts the inventory cycle, where T denotes the cycle's period. By using this notation, the impact of vendor/manufacturer and buyer holding costs as well as the proportion of defective items would be assessed on the yearly supply cost.

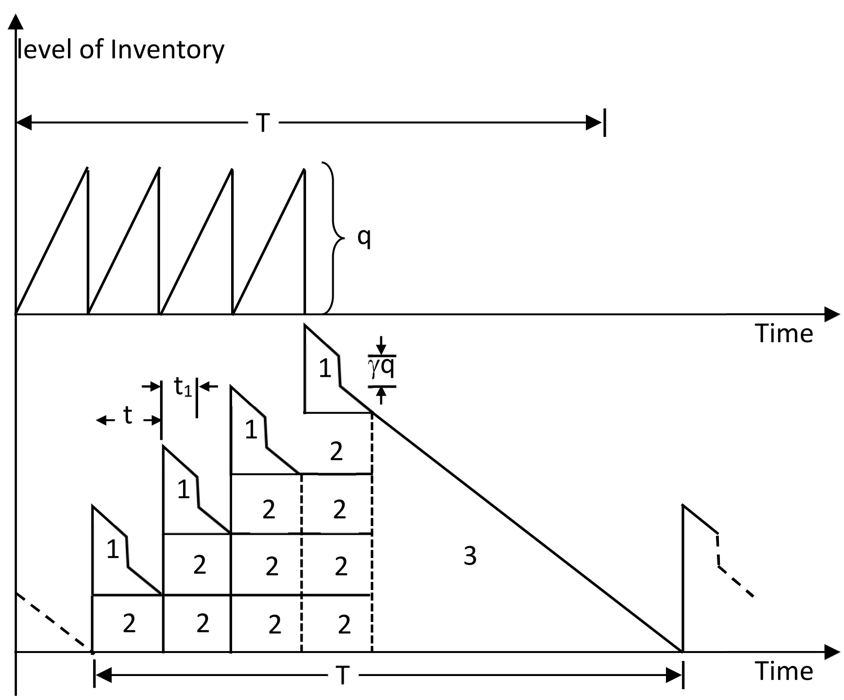

Fig. 1: The inventory cycle

The area of "1" based on the estimated value of the defectives proportion.

$$\text{Area } 1 = \frac{1}{2}\left(\frac{q}{p}\right)\left(\frac{\lambda D q}{p}\right) + (\lambda\gamma q)\left(\frac{q}{x}\right) = \frac{\lambda D q^2}{2p^2} + \frac{\lambda\gamma q^2}{x} \quad (1)$$

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$$\text{Area 2} = \frac{\lambda(\lambda+1)}{}\left(\frac{q}{2*p}\right)\left[-\frac{Dq}{p}+(1-\gamma)q\right] = \frac{\lambda(1+\lambda)q^2}{2p}\left(-\gamma+1-\frac{D}{p}\right) \tag{2}$$

triangle '3' area is

or $$\text{Area 3} = \frac{\lambda^2 q^2}{2}\left(\frac{1-\gamma}{D}-\frac{1}{p}\right)\left(1-\gamma-\frac{D}{p}\right) \tag{3}$$

As a result, the regions in (1), (2), and (3) are used to calculate the purchaser's anticipated holding cost for the vendor/single producer's cycle of production.

$$HC_b = h_b''\lambda q^2\left(\frac{D}{2p^2}\right)+\frac{h_b''q^2}{2}\left(1-\gamma-\frac{D}{p}\right)\left\{\frac{\lambda(\lambda-1)}{p}+\lambda^2\left(\frac{1-\gamma}{D}-\frac{1}{p}\right)\right\} \tag{4}$$

: if $\lambda = 1$, $h_b'' = h$, $P = \infty$, the holding cost as perEq.(5) of [18].

$$AC_b = \lambda A_b + \gamma qd + h_b''q^2\left(\frac{\lambda D}{2p^2}+\frac{\lambda\gamma}{x}\right)$$

$$+\frac{h_b''q^2}{2}\left(-\gamma+1-\frac{D}{p}\right)\left\{\frac{\gamma(\gamma+1)}{p}+\gamma^2\left(\frac{-(\gamma-1)}{D}-\frac{1}{p}\right)\right\} \tag{5}$$

producer's cost which is expected in a cycle, would be

$$AC_v = A_v + \frac{\lambda h_v q^2}{2p} + h_v'\left[\lambda q^2\left(\frac{D}{2p^2}+\frac{\gamma}{x}\right)\right.$$

$$\left.+\frac{q^2}{2}\left(1-\gamma-\frac{D}{p}\right)\left\{\frac{\gamma(\gamma+1)}{p}+\gamma^2\left(\frac{1-\gamma}{D}-\frac{1}{p}\right)\right\}\right] \tag{6}$$

$$TC = A_v + \lambda A_b + d\lambda q + \frac{\lambda h_v q^2}{2p} + (h_b''+h_v')q^2\left[\gamma\left(\frac{D}{2p^2}+\frac{\gamma}{x}\right)\right.$$

$$\left.+\frac{1}{2}\left(1-\gamma-\frac{D}{p}\right)\left\{\frac{\lambda(\lambda+1)}{p}+\lambda^2\left(\frac{1-\lambda}{D}-\frac{1}{p}\right)\right\}\right]$$

For a udf for the probability of damaged items,

$$E[TC] = A_v + \lambda A_b + d\lambda q + \frac{\lambda h_v q^2}{2p} + (h_b''+h_v')\left[\lambda\left(\frac{q^2 D}{2p^2}+\frac{q^2 E[\gamma]}{x}\right)\right.$$

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$$+\frac{1}{2}\left(1-\frac{D}{p}-E[\gamma]\right)\left\{\frac{\lambda(1+\lambda)}{p}+\lambda^2\left(\frac{-E[\gamma]+1}{D}-\frac{1}{p}\right)\right\}\Bigg] \tag{7}$$

The cycle length is

$$E[T]=\frac{\lambda(1-E[\gamma])q}{D} \tag{8}$$

$$E[ATC]=\frac{M_2}{q}\left\{\frac{A_v}{\lambda}+A_b\right\}+M_2dD+\frac{h_vDM_2q}{2p}+(h''_b+h'_v)M_2qD\Bigg[\left(\frac{D}{2p^2}+\frac{M_1}{x}\right)$$

$$+\frac{1}{2}\left(1-M_1-\frac{D}{p}\right)\left\{\frac{(\lambda+1)}{p}+\lambda\left(\frac{1-M_1}{D}-\frac{1}{p}\right)\right\}\Bigg] \tag{9}$$

where $M = E[\gamma]$ and $M_2 = \frac{D}{1-E[\gamma]}$

$$q=\sqrt{\frac{\frac{A_v}{\lambda}+A_b}{\frac{h_v}{2p}+(h'_v+h''_b)\left[\left(\frac{D}{2p^2}+\frac{M_1}{x}\right)+\frac{1}{2}\left(1-M_1-\frac{D}{p}\right)\left\{\frac{(\lambda+1)}{p}+\lambda\left(\frac{1-M_1}{D}-\frac{1}{p}\right)\right\}\right]}}$$

(10)

The following procedure would be used to the above formula to get an ideal number of lots each batch ():

1. Presume$\lambda = 1$ and cost = ∞.
2. Calculatethe lot size
3. Determine the supplier chain's yearly cost. If E[ATC] < cost, set $\lambda = \lambda + 1$ and repeat steps 1 through 4. Otherwise stop

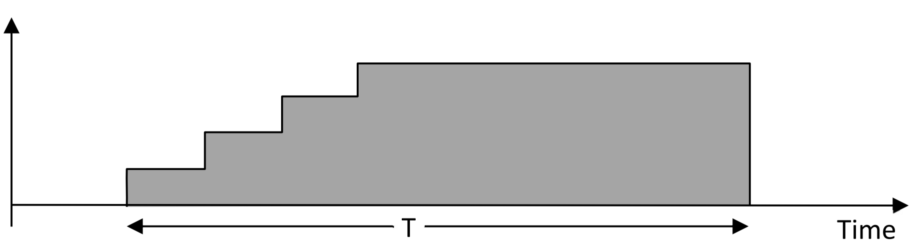

Fig. 2: Damagesinspectedby purchaser in a cycle

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$$E[TCU(q,\lambda)] = \frac{DM_2}{ }\left(\frac{A_v}{q\lambda} + 1/q(A_b)\right) + dDM_2$$

$$+\frac{qM_2}{2}\left[\frac{h_v D}{p} + (h'_v + h_b)\left\{(M_1+1)\left(\lambda + \frac{D}{p}\right) + D\left(\frac{2M_1}{x} - \frac{\lambda}{p}\right)\right\}\right]$$

$$+\frac{(s_b + h_b + h'_v)}{2} M_1 \left\{2\lambda q - \frac{qDM_2}{p}(\lambda - 1) - \frac{2D^2 M_2}{x}\right\} \quad (11)$$

$$E[TCU(q,\lambda)] = \frac{D}{q}\left(\frac{A_v}{\lambda} + A_b\right) + dD$$

$$+\frac{q}{2}\left[\frac{h_v D}{p} + (h_b + h'_v)\left\{(1 - M_1)\left(\lambda + \frac{D}{p}\right) + D\left(\frac{2M_1}{x} - \frac{\lambda}{p}\right)\right\}\right]$$

$$+\frac{(r_b + h_b + h'_v)}{2} M_1 \left\{2\lambda q - \frac{qD}{p}(\lambda - 1) - \frac{2D^2}{x}\right\} \quad (12)$$

Mathematical model:

2 models that integrate inventory calculations for SC participants are covered in this section.

The system's typical inventory is computed as

$$I_{cs} = \sum_{j=1}^{y} qrj\left(\frac{n_j}{2} - \frac{n_j d_{rj}}{2p_r} + \frac{d_{rj}}{p_r}\right) + d_{rj} l_j.$$

Ai $\sum_{j=1}^{y} q_{rj}^2 \left(\frac{n_j}{2p_r} - \frac{n_j^2}{2p_r} + \frac{n_j^2}{2d_{rj}}\right)$ by the cycle time $T_r = \sum_{j=1}^{y} \frac{n_j q_{rj}}{d_{rj}}$, the

$$I_{buyer} = \sum_{j=1}^{y} \frac{q_{rj}^2\left(\frac{n_j}{2p_r} - \frac{n_j^2}{2p_r} + \frac{n_j^2}{2d_{rj}}\right)}{T_r} = \sum_{j=1}^{y} \frac{d_{rj}}{n_j q_{rj}} \times q_{rj}^2 \left(\frac{n_j}{2p_r} - \frac{n_j^2}{2p_r} + \frac{n_j^2}{2d_{rj}}\right)$$

$$I_{transit} = \sum_{j=1}^{y} n_j q_{rj} l_j \times \frac{1}{T_r} = \sum_{j=1}^{y} \frac{d_{rj}}{n_j q_{rj}} \times n_j q_{rj} l_j = \sum_{j=1}^{y} d_{rj} l_j$$

$$I_{vendor}^{c1} = I_{cs} - I_{buyer} - I_{transit} = \sum_{j=1}^{y} \frac{q_{rj} d_{rj}}{2p_r}$$

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On an online channel, the typical inventory of essential items that require customization is estimated as

$$I_{vendor}^{c2} = \frac{q_c}{2} - \frac{q_c d_c}{2p_c}$$

(Offline channel)

Setup cost

$$\sum_{j=1}^{y} \frac{S_{mr} d_{rj}}{n_j q_{rj}}$$

Physical and financial holding cost:

$$\sum_{j=1}^{y} h_m^{pf} \frac{q_{rj} d_{rj}}{2p_r}$$

Financial holding cost:

The manufacturer's or vendor's expense for keeping such products in the buyers' godown is

$$\sum_{j=1}^{y} h_{mj}^{f} \left(\frac{n_j q_{rj}}{2} - (n_j - 1) \frac{q_{rj} d_{rj}}{2p_r} \right)$$

Production cost:

$$\sum_{j=1}^{y} p_{cr} \frac{d_{rj}}{p_r}$$

Transportation cost:

i

$$\sum_{j=1}^{y} \frac{n_j d_{rj}}{} \left((t_j / q_{rj}) + \upsilon_j \left(\frac{q_{rj}}{q_{rj} n_j} \right) \right) s$$

Lead time:

$$\sum_{j=1}^{y} \frac{d_{rj}}{q_{rj}} B(l_j)$$

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The entire inventory cost function relating to the channel which is offline is created by adding all of the afore mentioned costs.

$$C_m^{off}(n_j, s_{rj}, q_{rj}, l_j) = \sum_{j=1}^{y}\left[\frac{S_{mr}d_{rj}}{n_j q_{rj}} + h_m^{pf}\frac{q_{rj}d_{rj}}{2p_r} + h_{mj}^{f}\left(\frac{n_j q_{rj}}{2} - (n_j - 1)\frac{q_{rj}d_{rj}}{2p_r}\right) + p_{cr}\frac{d_{rj}}{p_r} + \frac{n_j d_{rj}}{q_{rj}}\left(t_j + \upsilon_j\left(\frac{q_{rj}}{n_j}\right)\right) + \frac{d_{rj}}{q_{rj}}B(l_j) + \left(h_{dj}^{p} + h_{mj}^{f}\right)d_{rj}l_j\right] \quad (13)$$

$$P_m^{off}(n_j, s_{rj}, q_{rj}, l_j) = \sum_{j=1}^{y}\left[w_{rj}d_{rj} - \left(\frac{S_{mr}d_{rj}}{n_j q_{rj}} + h_m^{pf}\frac{q_{rj}d_{rj}}{2p_r} + h_{mj}^{r}\left(\frac{n_j q_{rj}}{2} - (n_j - 1)\frac{q_{rj}d_{rj}}{2p_r}\right) + p_{cr}\frac{d_{rj}}{p_r} + \frac{n_j d_{rj}}{}\left((t_j / q_{rj}) + \upsilon_j\left(\frac{q_{rj}}{q_{rj}n_j}\right)\right) + \frac{d_{rj}}{q_{rj}}B(l_j) + \left(h_{dj}^{p} + j_{mj}^{f}\right)d_{rj}l_j\right)\right] \quad (14)$$

Setup cost for tailored items:

$$\frac{S_{mc}d_c}{q_c}$$

Holding cost for core items:

$$h_m^{pf}\left(\frac{q_c}{2} - \frac{q_c d_c}{2p_c}\right)$$

Production cost:

$$\sum_{i=1}^{z} p_{cci}\psi_{ci}\frac{d_c}{p_c}$$

$$C_m^{on}(s_{ci}, q_c) = \frac{S_{mc}d_c}{q_c} + h_m^{pf}\left(\frac{q_c}{2} - \frac{q_c d_c}{2p_c}\right) + \sum_{i=1}^{z} p_{cci}\psi_{ci}\frac{d_c}{p_c} \quad (15)$$

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The cost derived from function (3) is subtracted from customized products selling price which is paid by the online consumer to determine the vendor's or producer's profit using the online channel.

$$P_m^{on}(s_{ci}, q_c) = \sum_{i=1}^{z} s_{ci}\psi_{ci}d_c - \left(\frac{S_{mc}d_c}{q_c} + h_m^{pf}\left(\frac{q_c}{2} - \frac{q_c d_c}{2p_c}\right) + \sum_{i=1}^{z} p_{cci}\psi_{ci}\frac{d_c}{p_c}\right) \quad (16)$$

Ordering cost:

$$\sum_{j=1}^{y} O_{rj}\frac{d_{rj}}{q_{rj}}$$

Purchasing cost:

$$\sum_{j=1}^{y} w_{rj}d_{rj}$$

Physical holding cost:

$$\sum_{j=1}^{y} h_{rj}^{p}\left(\frac{n_j q_{rj}}{2} - (n_j - 1)\frac{q_{rj}d_{rj}}{2p_r}\right)$$

$$C_r^{off}(s_{rj}, q_{rj}) = \sum_{j=1}^{y}\left[O_{rj}\frac{d_{rj}}{q_{rj}} + w_{rj}d_{rj} + h_{rj}^{p}\left(\frac{n_j q_{rj}}{2} - (n-1)\frac{q_{rj}d_{rj}}{2p_r}\right)\right] \quad (17)$$

The profit of y purchasers $P_r^{off}(s_{rj}, q_{rj})$

$$P_r^{off}(s_{rj}, q_{rj}) = \sum_{j=1}^{y}\left[s_{rj}d_{rj} - \left(O_{rj}\frac{d_{rj}}{q_{rj}} + w_{rj}d_{rj} + h_{rj}^{p}\left(\frac{n_j q_{rj}}{2} - (n-1)\frac{q_{rj}d_{rj}}{2p_r}\right)\right)\right] \quad (18)$$

Model:

Until the purchaser's inventory touches its maximum pointthe vendor/manufacturer keeps making and distributing the standard item. With the jth purchaser taken into account, the SCintegrated profit is calculated.

$$P_{mrj}^{1}(n_j, s_{rj}, s_{ci}, q_{rj}, q_c, l_j) = \left[\left((1-\theta_s)a_d - \alpha_{rj}s_{rj} + \rho\sum_{i=1}^{z} s_{ci} - \beta_{rj}l_j\right)\right.$$

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$$\left[s_{rj} - \frac{(S_{mr} + n_j O_{rj})}{n_j q_{rj}} - h_m^{pf} \frac{q_{rj}}{2p_r} + \left(h_{mj}^{f} + h_{rj}^{p}\right)\left((n_j - 1)\frac{q_{rj}}{2p_r} \right) - \frac{p_{cr}}{p_r} \right.$$

$$\left. - \frac{n_j}{}\left(t_f / q_{rj} + \upsilon_j \left(\frac{1}{n_j} \right) \right) - \frac{B(l_j)}{q_{rj}} - \left(h_{dj}^{p} + h_{mj}^{f}\right) \right] - \left(h_{mj}^{r} + h_{rj}^{p}\right)\left(\frac{n_j q_{rj}}{2} \right)$$

$$+ \left(\theta_s a_d - \sum_{i=1}^{z} \alpha_{ci} s_{ci} + \rho s_{rj} + \beta_c l_j \right) \times \left[\sum_{i=1}^{z} s_{ci} \psi_{ci} - \frac{S_{mc}}{q_c} + h_m^{pf} \left(\frac{q_c}{2p_c} \right) \right.$$

$$\left. - \sum_{i=1}^{z} \frac{p_{cci} \psi_{ci}}{p_c} \right] - h_m^{pf} \left(\frac{q_c}{2} \right) \Bigg] \tag{19}$$

Consequently, the complete profit for one /manufacturer and y purchasers may be expressed as

$$P_{mr}^{1}(n_j, s_{rj}, q_c, l_j, s_{ci}, q_{rj},) = \sum_{j=1}^{y} P_{mrj}^{1}(n_j, s_{rj}, q_c, l_j, s_{ci}, q_{rj},) \tag{20}$$

Since there is little information about market demand, a fuzzy optimization model has been created,the trapezoidal fuzzy number helps to eliminate demand uncertainty or a lack of knowledge.Consequently, the following are the ways in which the fuzzification and defuzzification procedure is applied:

Fuzzification:

On an offline channel, the demand ratio among y buyers is given as

$$\tilde{d}_{rj} = (d_{rj} - \varphi_{drj1}, d_{rj} - \varphi_{drj2}, d_{rj} + \varphi_{drj3} d_{rj} + \varphi_{drj4})$$

and the rate of demand in online

$$\tilde{d}_{c} = (d_c - \varphi_{dc1}, d_c - \varphi_{dc2}, d_c + \varphi_{dc3}\ d_c + \varphi_{dc4})$$

Thus, given the following circumstances, j = 1, 2, 3, and drj1, drj2, drj3, drj4, dc1, dc2, dc3, and dc4 are arbitrary positive numbers: DRC > DRC1 > DRC2 > DRC3 > DRC4; DC > DRC1 > DRC2 > DRC3 > DRC4. As a result, the total profit for one vendor/manufacturer and one buyer is as follows:

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$$\tilde{P}^2_{mrj}(n_j, q_{rj}, q_c, l_j) = \left[\sum_{i=1}^{z} s_{ci}\psi_{ci}\tilde{d}_c + s_{rj}\tilde{d}_{rj} - \left((S_{mr} + n_j O_{rj})\frac{\tilde{d}_{rj}}{n_j q_{rj}} + \frac{S_{mc}\tilde{d}_c}{q_c} + h_m^{pf}\frac{q_{rj}\tilde{d}_{rj}}{2p_r} \right.\right.$$

$$+\left(h^f_{mj} + h^p_{rj}\right)\left(\frac{n_j q_{rj}}{2} - (n_j - 1)\frac{q_{rj}\tilde{d}_{rj}}{2p_r}\right) + h_m^{pf}\left(\frac{q_c}{2} - \frac{q_c\tilde{d}_c}{2p_c}\right) + p_{cr}\frac{\tilde{d}_{rj}}{2p_r}$$

$$\left.\left. + \sum_{i=1}^{z} p_{cci}\psi_{ci}\frac{\tilde{d}_c}{p_c} + \frac{n_j\tilde{d}_{rj}}{q_{rj}}\left(t_j + \upsilon_j\left(\frac{q_{rj}}{n_j}\right)\right) + \frac{\tilde{d}_{rj}}{q_{rj}}B(l_j) + \left(h^p_{dj} + h^r_{mj}\right)\tilde{d}_{rj} l_j \right)\right]$$

As a result, the following is the entire profit fuzzified for a producer and y buyers:

$$\tilde{P}^2_{mr}(n_j, q_c, l_j, q_{rj}) = \sum_{j=1}^{y} \tilde{P}^2_{mrj}(n_j, q_c, l_j, q_{rj}) \qquad (21)$$

Defuzzification:

Defuzzification is the method forusing fuzzy sets and associated membership functions to produce a quantified output in crisp logic (i.ethe process of transforming a fuzzy amount into a crisp quantity or a fuzzy set into a crisp set). The drj and dc arc's left and right cuts are provided below

$$\tilde{d}_{Lrj}(\lambda) = d_{rj} - \varphi_{drj1} + (\varphi_{drj1} - \varphi_{drj2})\lambda;$$

$$\tilde{d}_{Urj}(\lambda) = d_{rj} - \varphi_{drj4} + (\varphi_{drj4} - \varphi_{drj3})\lambda$$

and $$\tilde{d}_{Lc}(\lambda) = d_c - \varphi_{dc1} + (\varphi_{dc1} - \varphi_{dc2})\lambda;$$

$$\tilde{d}_{Uc}(\lambda) = d_c - \varphi_{dc4} + (\varphi_{dc4} - \varphi_{dc3})\lambda;$$

profit function which is the integrated with left and right λ cuts $\tilde{P}^2_{mr}(n_j, q_{rj}, q_c, l_j)$ (9) are

$$\tilde{P}^2_{mr}(n_j, q_{rj}, q_c, l_j)L(\lambda) = \sum_{j=1}^{y}\left[\sum_{i=1}^{z} S_{ci}\psi_{ci}\tilde{d}_{Lc}(\lambda) + S_{rj}\tilde{d}_{Lrj}(\lambda) \right.$$

$$-\left((S_{mr} + n_j O_{rj})\frac{\tilde{d}_{Lrj}(\lambda)}{n_j q_{rj}} + \frac{S_{mc}\tilde{d}_{Lc}(\lambda)}{q_c} + h_m^{pf}\frac{q_{rj}\tilde{d}_{Lrj}(\lambda)}{2p_r} \right.$$

$$+\left(h^f_{mj} + h^p_{rj}\right)\left(\frac{n_j q_{rj}}{2} - (n_j - 1)\frac{q_{rj}\tilde{d}_{Lrj}(\lambda)}{2p_r}\right) + h_m^{pr}\left(\frac{q_c}{2} - \frac{q_c\tilde{d}_{Lc}(\lambda)}{2p_c}\right)$$

$$+p_{cr}\frac{\tilde{d}_{Lrj}(\lambda)}{q_{rj}}+\sum_{i=1}^{z}p_{cci}\psi_{ci}\frac{\tilde{d}_{Lc}(\lambda)}{p_c}+\frac{n_j\tilde{d}_{Lrj}(\lambda)}{q_{rj}}\times\left(t_j+\upsilon_j\left(\frac{q_{rj}}{n_j}\right)\right)$$

$$\left.\left.+\frac{\tilde{d}_{Lrj}(\lambda)}{q_{rj}}B(l_j)+\left(h_{dj}^{p}+h_{mj}^{f}\right)\tilde{d}_{Lrj}(\lambda)l_j\right)\right]$$

and $$\tilde{P}_{mr}^{2}(n_j,q_{rj},q_c,l_j)U(\lambda)=\sum_{j=1}^{y}\left[\sum_{i=1}^{z}S_{ci}\psi_{ci}\tilde{d}_{Uc}(\lambda)+S_{rj}\tilde{d}_{Urj}(\lambda)\right.$$

$$-\left((S_{mr}+n_jO_{rj})\frac{\tilde{d}_{Urj}(\lambda)}{n_jq_{rj}}+\frac{S_{mc}\tilde{d}_{Uc}(\lambda)}{q_c}+h_{m}^{pf}\frac{q_{rj}\tilde{d}_{Urj}(\lambda)}{2p_r}\right.$$

$$+\left(h_{mj}^{f}+h_{rj}^{p}\right)\left(\frac{n_jq_{rj}}{2}-(n_j-1)\frac{q_{rj}\tilde{d}_{Urj}(\lambda)}{2p_r}\right)+h_{m}^{pr}\left(\frac{q_c}{2}-\frac{q_c\tilde{d}_{Uc}(\lambda)}{2p_c}\right)$$

$$+p_{cr}\frac{\tilde{d}_{Urj}(\lambda)}{q_{rj}}+\sum_{i=1}^{z}p_{cci}\psi_{ci}\frac{\tilde{d}_{Uc}(\lambda)}{p_c}+\frac{n_j\tilde{d}_{Urj}(\lambda)}{q_{rj}}\times\left(t_j+\upsilon_j\left(\frac{q_{rj}}{n_j}\right)\right)$$

$$\left.\left.+\frac{\tilde{d}_{Urj}(\lambda)}{q_{rj}}B(l_j)+\left(.h_{dj}^{p}+h_{mj}^{f}\right)\tilde{d}_{Urj}(\lambda)l_j\right)\right]$$

respectively. In order to change the signed distance technique to a crisp one, the profit function's fuzzification to a trapezoidal fuzzy number may be similar.

$$d_O\left(\tilde{P}_{mr}^{2}(n_j,q_{rj},q_c,l_j),\tilde{O}\right)=\frac{1}{2}\int_0^1\left[\tilde{P}_{mr}^{2}(n_j,q_{rj},q_c,l_j)L(\lambda)+\tilde{P}_{mr}^{\,2}(n_j,q_c,l_j,q_{rj}),U(\lambda)\right]d\lambda$$

Defuzzified profit function the supply chain's is

$$d_O\left(\tilde{P}_{mr}^{2}(n_j,q_{rj},q_c,l_j),\tilde{O}\right)=\sum_{j=1}^{y}\left[\sum_{i=1}^{z}s_{ci}\psi_{ci}d_O\left(\tilde{d}_c,\tilde{O}\right)+s_{rj}d_O\left(\tilde{d}_{rj},\tilde{O}\right)\right.$$

$$-\left((S_{mr}+n_jO_{rj})\frac{d_O\left(\tilde{d}_{rj},\tilde{O}\right)}{n_jq_{rj}}+\frac{S_{mc}d_O\left(\tilde{d}_c,\tilde{O}\right)}{q_c}+h_{m}^{pf}\frac{q_{rj}d_O\left(\tilde{d}_{rj},\tilde{O}\right)}{2p_r}\right.$$

$$+\left(h_{mj}^{f}+h_{rj}^{p}\right)\left(\frac{n_jq_{rj}}{2}-(n_j-1)\times\frac{q_{rj}d_O\left(\tilde{d}_{rj},\tilde{O}\right)}{2p_r}\right)+h_{m}^{pr}\left(\frac{q_c}{2}-\frac{q_cd_O\left(\tilde{d}_c,\tilde{O}\right)}{2p_c}\right)$$

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$$+p_{cr}\frac{d_O\left(\tilde{d}_{rj},\tilde{O}\right)}{p_r}+\sum_{i=1}^{z}p_{cci}\psi_{ci}\times\frac{d_O\left(\tilde{d}_{rj},\tilde{O}\right)}{q_{rj}}B(l_j)+\left(h_{mj}^{f}+h_{dj}^{p}\right)d_O\left(\tilde{d}_{rj},\tilde{O}\right)l_j \quad (22)$$

where $d_O=\left(\tilde{d}_{rj},\tilde{O}\right)=d_{rj}+\frac{1}{4}(\varphi_{drj4}+\varphi_{drj3}-\varphi_{drj2}-\varphi_{drj1})>0$ (23)

$$d_O=\left(\tilde{d}_c,\tilde{O}\right)=d_c+\frac{1}{4}(\varphi_{dc4}+\varphi_{dc3}-\varphi_{dc2}-\varphi_{dc1})>0 \quad (24)$$

The discovered defuzzified profit function for one supplier/producer and jth purchaser is

$$d_O\left(\tilde{P}_{mrj}^{2}(n_j,q_{rj},q_c,l_j),\tilde{O}\right)=\Bigg[\left(d_{rj}+\frac{1}{4}(\varphi_{drj4}+\varphi_{drj3}+\varphi_{drj2}+\varphi_{drj1})\right)$$

$$\Bigg[s_{rj}-\frac{(S_{mr}+n_jO_{rj})}{n_jq_{rj}}-h_m^{pf}\frac{q_{rj}}{2p_r}+\left(h_{mj}^{f}+h_{rj}^{p}\right)\left((n_j-1)\frac{q_{rj}}{2p_r}\right)-\frac{p_{cr}}{p_r}$$

$$-\frac{n_j}{q_{rj}}\left(t_j+\upsilon_j\left(\frac{q_{rj}}{n_j}\right)\right)-\frac{B(l_j)}{p_{rj}}-\left(h_{dj}^{p}+j_{mj}^{f}\right)l_j\Bigg]-\left(h_{mj}^{f}+j_{rj}^{p}\right)\left(\frac{n_jq_{rj}}{2}\right)$$

$$+\left(d_c+\frac{1}{4}(\varphi_{dc4}+\varphi_{dc3}-\varphi_{dc2}-\varphi_{dc1}\right)\times\Bigg[\sum_{i=1}^{z}s_{ci}\psi_{ci}-\frac{S_{mc}}{q_c}+h_m^{pf}\left(\frac{q_c}{2p_c}\right)$$

$$-\sum_{i=1}^{z}\frac{p_{cci}\psi_{ci}}{p_c}\Bigg]-h_m^{pf}\left(\frac{q_c}{2}\right)\Bigg] \quad (25)$$

Solution procedure:

Algorithms:

Two strategies are provided in this area for quantitatively assessing the suggested methods

Algorithm-I

Step-1:let$[n_1, n_2, n_3, \dots, n_y] = [1, 1, 1, \dots, 1]$.

Step-2: For each [l1, l2, l3,..., ly], repeat the subsequent steps from 2.1 to 2.4.

Step-2.1: Calculate [sr1, sr2, sr3,..., sry] using Eq. (16) given the variables [sc1, sc2, sc3, sc4..., scz], [qr1, qr2, qr3,..., qry] and qc e started.

Step-2.2: Calculate [sc1, sc2, sc3,..., scz] using Eq. using the [sr1, sr2, sr3,..., sry], [qr1, qr2, qr3,..., qry] ,qc (19).

Step-2.3: Calculate [qr1, qr2, qr3,..., qry] and [sr1, sr2, sr3,..., sry], [sc1, sc2, sc3,..., scz], qc and [sr1, sr2, sr3,..., sry], [sc1, sc2, sc3.. (25).

Step-2.4: Steps 2.1 through 2.3 should be repeated until [sr1, sr2, sr3,..., sry], [sc1, sc2, sc3,..., scz], [qr1, qr2, qr3,..., qry], and qc have the same values.

Step-3: tusing[sr1, sr2, sr3,..., sry], [sc1, sc2, sc3,..., scz], [qr1, qr2, qr3,..., qry] and qc, computethe supply chain's total profit (8).

Steps 2 through 3 are repeated, setting nj = nj + 1.

Step-4: If and only if j = 1, 2, 3,..., y.

The best answer is [n1, n2, n3,..., ny], [sr1, sr2, sr3,..., sry], [sc1, sc2, sc3,..., scz], [qr1, qr2, qr3,..., qry], qc, and [l1, l2, l3,..., ly].

Algorithm-II

Step-1: Set $[n_1, n_2, n_3, \ldots, n_y] = [1, 1, 1, \ldots, 1]$.

Step-2: For every $[l_1, l_2, l_3, \ldots, l_y]$ (see Table-1) and follow the subsequent steps from 2.1 to 2.3.

Step-2.1: Initialiazation of qc values, then use Eq. (28) to calculate [qr1, qr2, qr3,..., qry].

Step-2.2:using [qr1, qr2, qr3,..., qry] variables and Eq. (31) to determine qc

.**Step-2.3:**Replicate steps 2.1 to 2.2 until there is no change in the estimates of [qr1, qr2, qr3,..., qry] and qc.

Step-3:utilizing [qr1, qr2, qr3,..., qry] variables and Eq. (31) to determine qc

Step-4:let$n_j = n_j + 1$, perform Steps from 2 to 3.

Step-5: If $d_0\left(\tilde{p}^2_{mrj}(n_j, q_{rj}, q_c, l_j), \tilde{0}\right) \geq d_0\left(\tilde{P}^2_{mrj}(n_j+1, q_{rj}, q_c, l_j), \tilde{0}\right)$ and $d_0\left(\tilde{P}^2_{mrj}(n_j, q_{rj}, q_c, l_j), \tilde{0}\right) \geq d_0\left(\tilde{p}^2_{mrj}(n_j-1, q_{rj}, q_c, l_j), \tilde{0}\right)$ wherever j = 1,2,3,...,y.

Step-6: $d_0(\tilde{P}^2_{mr}(n^*_j, q^*_{rj}, q^*_c, l^*_j), \tilde{O}) = \max d_0(\tilde{P}^2_{mr}(n^*_j, q^*_{rj}, q^*_c, l^*_j), \tilde{O})$ and we obtain $[n_1, n_2, n_3, \ldots, n_y]$, $[q_{r1}, q_{r2}, q_{r3}, \ldots, q_{ry}]$, q_c and $[l_1, l_2, l_3, \ldots, l_y]$ as the feasible bestsolution.

Table-1: Components of Lead time

Purchaser j	Lead time components k	Normal time$n_{j,k}$(annually)	Smallest time m_{jk} (years)	Unit cost $e_{j,k}$ (rs/year)
1	1	20/344 = 0.04479	4/344 = 0.01444	0.1 × 344 = 34.4
	2	20/344 = 0.04479	4/344 = 0.01444	1.2 × 344 = 438

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	3	14/344 = 0.04383	9/344 = 0.02444	4.0 × 344= 1824
2	1	20/344 = 0.04479	4/344 = 0.01444	0.4 × 344= 182.4
	2	14/344 = 0.04383	9/344 = 0.02444	1.3 × 344 = 474.4
	3	13/344 = 0.034414	4/344 = 0.01444	4.1 × 344= 1841.4
3	1	24/344 = 0.04849	11/344 = 0.03013	0.4 × 344 = 144
	9	20/344 = 0.04479	4/344 = 0.01444	2.4 × 344 = 912.4
	3	18/344 = 0.04931	11/344=0.03013	4.0 × 344= 1824

Table-2: Lead time summary

Purchaser j	Lead time l_j (years)	B(l_j) (rs/shipment)
1	46/364 = 0.14342	0
	42/364 = 0.11406	1.4
	28/364 = 0.076712	18.2
	21/364 = 0.04743	43.20
7	49/364 = 0.13424	0
	34/364 = 0.094890	7
	28/364 = 0.076712	16.1
	21/364 = 0.047434	41.8
3	63/364 = 0.1726	0
	49/364 = 0.13424	4.6
	34/364 = 0.09489	40.6
	28/364 = 0.076712	74.6

Source:Data are taken from Ben-Daya [5] and Batarfi et al. [7].

Data set-1: Parameters associated with vendor/manufacturer: S_{mr} = 1000 (rs/setup), S_{mc} = 800 (rs/setup), P_r = 18,000 (rs/annually), p_c = 18,000 (rs/year), p_{cr} = 500,000 (rs/year), p_{cci} = 750,000 (rs/year), Ψ_{ci}= 1 (%), h_m^{pf} = 30 (rs/unit/year). Parameters concernedwith purchasers: [w_{r1}, w_{r3}, w_{r3}] = [300, 300, 300] (rs/unit), [O_{r1}, O_{r3}, O_{r3}] = [300, 300, 350] (rs/order), $\left[h_{r2}^p, h_{r2}^p, h_{r3}^p\right]$ = [10,6, 8] (rs/unit/year), $\left[h_{m1}^f, h_{m2}^f, h_{m3}^f\right]$= [30, 10, 15] (rs/unit/year), $\left[h_{d1}^p, h_{d2}^p, h_{d3}^p\right]$= [10,5,7] (rs/unit/year),

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$[t_1, t_3, t_3]$= [30,10,15] (rs/shipment), $[\upsilon_1, \upsilon_3, \upsilon_3]$ =[5, 4, 3] (rs/unit), parameters: a_d = 15,000 (units/year), θ_s = 0.3(%), ρ = 1.8, α_{ci} = 3 (unit^3/rs/year), β_c = 50 (customerdaily), $[\alpha_{r1}, \alpha_{r3}, \alpha_{r3}]$ = [30, 10, 15] (unit^3/rs/year), $[\beta_{r1}, \beta_{r3}, \beta_{r3}]$ = [40, 30, 30] (customer/ day). Aside from. Table 3 provides the lead time components for the three consumers with usual and lowest durations based on the cost of each component to crash. Additionally, Table-3 provides a summary of each buyer's controllable lead time and the corresponding crashing cost.

Data set-2 Fuzzy parametric values: $[d_{r1}. d_{r2}, d_{r3}]$ = [4000, 4000, 4400] (unit/year), $[\varphi d_{r11}, \varphi d_{r21}, \varphi d_{r31}]$= [2400,2000,2240], $[\varphi d_{r12}, \varphi d_{r22}, \varphi d_{r32}]$ = [1240,1000, 1124], $[\varphi d_{r13}, \varphi d_{r23}, \varphi d_{r33}]$= [1240, 1000. 1124], $[\varphi d_{r14}, \varphi d_{r24}, \varphi d_{r34}]$ = 12400,2000, 2240], d_c=3400 (unit/year),φd_{c1} = 1740, φd_{c2}= 874, φd_{c3} = 874,φd_{c4} = 1740.

Numerical discussion:

As stated by, it is expected that the percentage of defectives is distributed equally.

$$f(\gamma) = \begin{cases} 25, & 0 \le \gamma \le 0.04 \\ 0, & \text{otherwise} \end{cases}$$

The findings demonstrate that the Salameh and Jaber's [18] technique and the consignment stock policy (CS) put forward by Braglia and Zavanella [3] can be coupled to reduce the supply chain's annual cost. The expenditures of inspection and of holding the damaged items are the main causes of this discrepancy. The behaviour of the models used in this paper and by Khan et al. [35] is depicted in Fig. 3. It should be noted that the realistic methodology presented in this study would induce a reduction in both the supply chain's overall cost and the percentage of defective items. Additionally, the difference in storage expenses between the 2 stakeholders largely determines how much the cost will move.

D Units/yr	**P Units/yr**	**A_v rs/cycle**	**A_b rs/cycle**	**h'_v rs/unit/yr**	**h''_v rs/unit/yr**	**h'_b rs/unit/yr**	**h''_b rs/unit/yr**	**D rs/unit**	**X Unit/yr**
1000	**3200**	**400**	**25**	**2**	**6**	**3**	**2**	**0.6**	**175200**
				# of lots per cycle		**Quantity per lot**		**Assumed annual cost**	
				4		**113**		**2409**	

Each buyer shipped products totaling respectively [382.71, 809.26, 495.59] and [385.83, 419.34, 401.57]. In the initial model, the number of shipments (nj) determines how many core items are manufactured for customisation (qc), hence qc's value fluctuates with nj's values. Because the quantity of shipments nj will have an impact on the shipment size for three customers (i.e., qr1, qr2, and qr3), these two models will alter as a result. The manufacturing quantities of the basic components for Models I and II are 368.35 and 481.374, respectively. The qc remains linear with regard to the numbers of nj (nj) in the second model, which does not depend on the volume of shipments.

Table-3: Feasible values

Example	Purchaser j	l_j	n_j	s_{rj}	q_{rj}	s_{ci}	q_c	P_{nrj}	Total	Profit (%)
1	1	0.04743	2	414.23	382.71			4112042.9		
	2	0.04743	2	809.26	498.49	1404.88	369.34	6699477.0	17723826.0	22.5185
	3	0.076712	2	444.28	494.49			4912204.7		
2	1	0.04743	2		384.83			6896864.0		
	2	0.04743	2	–	419.34	–	481.37	8147246.4	22221374.0	
	3	0.076712	2		401.47			7177263.4		

In Data set-1, $P_{mrj} = P^1_{mrj}$ = profit of jth purchaser of the supply chain & profit P^1_{mr} (n_j, s_{rj}, s_{ci}, q_{rj}, q_c, l_j). In Data set-2, $P_{mrj} = d_0\left(\tilde{P}^2_{mrj}, \tilde{0}\right)$ = profit of j^{th}purchaser of the supply chain & profit = $d_0\left(\tilde{P}^2_{mrj}(n_j, q_{rj}, q_c, l_j), \tilde{0}\right)$.

$$\text{Profit percentage difference} = \frac{\left|P^1_{mr}(n_{j,}s_{rj}, q_{rj}, q_c, s_{ci}, l_j) - d_0\left(\tilde{P}^2_{mr}(n_j, q_c, l_j, q_{rj}), \tilde{0}\right)\right|}{\frac{\left(P^1_{mr}(n_{j,}s_{rj}, s_{ci}, q_c, q_{rj}, l_j) + d_0\left(\tilde{P}^2_{mr}(n_j, q_c, l_j, q_{rj}), \tilde{0}\right)\right)}{2}} \times 100$$

Because the profitability of each client in the second model seems to be higher than it was in the first. Through Model-s, the supply chain participants generated profits/gains totaling rs 17723826 and rs22221375, respectivelyAs a result, it demonstrates a 22. 185% difference in the profit percentage between the two models. This study examines the supply chain of one vendor/manufacturer and one buyer. The manufacturer or supplier is supposed to deliver a single item, but it's assumed that some of their lots have known defective products in them. To distinguish these substandard products, the buyer implements a 100% inspection process. According to the

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model, the cost increases dramatically as the percentage of defective items increases. The outcomes also demonstrated that our model's annual expenses are better.

The physical and monetaryholding expensesgains/reduces, the total profit $P^{1}_{mr}(n_{j,}s_{rj}, q_c, l_j, s_{ci}, q_{rj})$ and $d_0\left(\tilde{P}^{2}_{mr}(n_j, q_c, l_j, j), \tilde{0}\right)$ reduces/gains. Shows the order cost (O_{rj}) is inversely proportional to the profit $\left(P^{1}_{mr}(n_{j,}s_{rj}, q_{rj}, q_c, s_{ci}, l_j\right)$ and $d_0\left(\tilde{P}^{2}_{mr}(n_j, q_{rj}, q_c, l_j), \tilde{0}\right)\Big)$.

4. CONCLUSION

The demand rates for both channels are thought to be uncertain, and the fuzzy trapezoidal number is used to handle the uncertainty. The study makes an effort to optimise overall supply chain profit for supply chain participants (vendors, manufacturers, and clients) using a variety of strategies. According to the second model, which, in contrast to the first, shows the highest numerical advantage, the profit is increased when the product requirement is trapezoidal. This research's model includes an unusual innovation that takes environmental considerations like emissions and energy use in manufacturing and transportation into account.

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