Journal of Research Administration
Society of Research Administrators International

NEW ALPHA POWER INVERSE WEIBULL DISTRIBUTION WITH RELIABILITY APPLICATION ON THE TIME SPENT WAITING FOR ASSISTANCE AT TWO BANKS

Kurdistan M. Taher Omar

Department of Mathematics, Faculty of Science, University of Zakho, Kurdistan Region, Iraq

kurdistan.taher@uoz.edu.krd

Abstract: In this study, a novel generalized New Alpha Inverse Weibull (NAPIW) distribution is defined utilizing the Alpha power transformation method. Its statistical features, such as reliability and moments, are obtained. Additionally, the estimate of the NAPIW parameters using the maximum likelihood estimation approach is discussed. Eventually, the proposed new distribution is applied to actual data reflecting the time spent waiting for customer assistance in a bank, and its goodness-of-fit is shown. Furthermore, we use simulation data as well. Moreover, calculating mean square errors and bias for all parameters and some other measurements as well.

keywords: New Alpha Power Transformation; Inverse Weibull Distribution; Reliability; Moments; Maximum Likelihood Estimation; Simulation

1. Introduction

In many applications like the dynamic parts of diesel engines and different data sets like the times it takes for an insulating fluid under continual tension to break down, the inverse Weibull (IW) distribution is essential. For further information, see [1].

The Inverse Weibull Distribution with parameters a, b, and ρ specifies the Probability Density Function and Cumulative Distribution Function of a random variable x.

$$F(x) = \frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}},$$

and

$$f(x) = 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b+a}\ (tx)^2}\left(1-e^{-(tx)^2}\right)^{-\rho b-1}.$$

Inverse Weibull Distribution research has been extensively investigated, with [2] studying the Maximum Likelihood and Least Squares Estimation of the inverse Weibull distribution, for instance. Inverse Weibull Distribution Bayes 2 -sample prediction has been addressed by [3]. The Inverse Weibull Distribution has been fitted to the flood data for [4]. [5] conducted some significant theoretical research on the Inverse Weibull Distribution.

Several Inverse Weibull Distribution generalizations have recently been examined by authors, including [6]'s of two Inverse Weibull Distributions, the generalized Inverse Weibull Distribution by [7], the modified Inverse Weibull Distribution by [8], beta Inverse Weibull Distribution by [9], Gamma Inverse Weibull Distribution by [10], Inverse Weibull Distribution improved via Kumaraswamy by [11] by [12] considered generalized Beta Inverse Weibull Distribution, and dy [13], the Marshall-Olkin expanded Inverse Weibull distribution was discovered.

To produce a family of distributions, [14] suggested transforming the baseline (CDF) by including a new parameter. [15] pointed the applications of reliability with Alpha power inverse Weibull distribution such as hazard function, mean residual life, and moments. The suggested technique is referred to as New Alpha Power Transformation (NAPT) that is significantly differs from existing Alpha Inverse Weibull distributions by numbers of variables $(\mathrm{a}, \mathrm{b}, \rho)$ and time as well. The GAPT (X) is the cumulative density distribution function if $F(x)$ is a Cumulative Density Function of any distribution.

$$G_{APT}(X) = \begin{cases} \dfrac{\alpha^{F(x)} - 1}{\alpha - 1}, & \alpha > 0, \alpha \neq 1 \\ F(x), & \alpha = 1 \end{cases}$$

the related probability density function (PDF), as well as

$$g_{APT}(X) = \begin{cases} \dfrac{\log(\alpha)}{\alpha - 1} f(x)\alpha^{F(x)}, & \alpha > 0, \alpha \neq 1 \\ f(x), & \alpha = 1 \end{cases}$$

2. New Model

In this part, we introduce the New Alpha Power Inverse Weibull Distribution and several of its submodels.

2.1. NAPIW Specification

A new distribution known as the NAPIW (x, Θ) distribution is obtained by substituting the Cumulative Function of the inverse Weibull distribution provided by (1) in New Alpha Power with the distribution accorded by (3) when $\Theta = (a, b, \rho, \alpha,)$.

$$G_{NAPIW}(X) = \begin{cases} \dfrac{\alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}}} - 1}{\alpha - 1}, & \alpha > 0, \alpha \neq 1 \\ \dfrac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}}, & \alpha = 1. \end{cases}$$

It is provided with its matching Probability Density Function (PDF).

$$g_{NAPIW}(X) = \begin{cases} \dfrac{\log(\alpha)}{\alpha - 1} 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b + a \ (tx)^2}} \left(1 - e^{-(tx)^2}\right)^{-\rho b - 1} \\ \qquad \alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{}_{,\alpha>0,\alpha\neq 1}, \ \alpha = 1 \\ 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b + a - (tx)^2}} \left(1 - e^{-(tx)^2}\right)^{-\rho b - 1}, \ \alpha = 1. \end{cases}$$

By utilizing Taylor's series expansion of the function $e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}$, we can rewrite the PDF when $\alpha > 0, \alpha \neq 1$ as follows:

$$e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho}} = \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} \left[a\left(1 - e^{-(tx)^2}\right)^{-\rho b}\right]^s$$

$$= \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} a^s \left(1 - e^{-(tx)^2}\right)^{-\rho b s},$$

and

$$\alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}} = \sum_{j=0}^{\infty} \frac{\log(\alpha)^{j+1}}{j_{j!}},$$

then

$$g_{NAPIW}(X) = \frac{1}{\alpha - 1} \sum_{j=0}^{\infty} \frac{\log(\alpha)^{j+1}}{j!} 2ab\rho t^2 x \sum_{s=0}^{\infty} \sum_{d=0}^{\infty} \frac{(-1)^s a^s}{s_! d_!} \frac{\Gamma(abs\rho + b\rho + d + 1)}{\Gamma(bs\rho + b\rho + 1)},$$

$$e^{-(tx)^2} e^{-d(tx)^2}$$

$$g_{NAPIW}(X) = \frac{2ab\rho t^2 x}{\alpha - 1} \sum_{j=d=s=0}^{\infty} \frac{\log(\alpha)^{j+1} (-1)^s a^s \Gamma(abs\rho + b\rho + d + 1)}{j!\, s!\, d!\, \Gamma(bs\rho + b\rho + 1)},$$

$$e^{-(d+1)(tx)^2}$$

$$g_{NAPIW}(X) = \psi_{s,d,j} x e^{-(d+1)(tx)^2}.$$

Then

$$\psi_{s,d,j} = \frac{2ab\rho t^2}{\alpha - 1} \sum_{j=d=s=0}^{\infty} \frac{\log(\alpha)^{j+1}(-1)^s a^s \Gamma(abs\rho + b\rho + d + 1)}{j!\, s!\, d!\, \Gamma(bs\rho + b\rho + 1)}.$$

The visual illustration of PDF for the new model is displayed in Figure 1

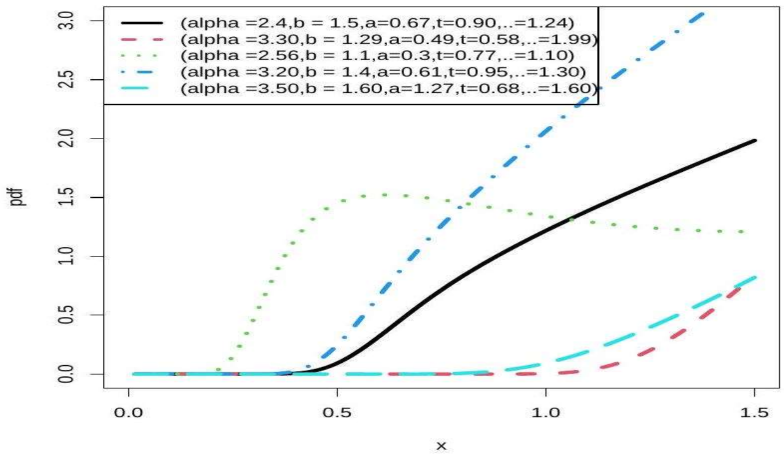

Figure 1: PDF plot for new model

3. Analysis of Reliability

The survival analysis (Reliability Function) of the NAPIW Distribution is provided by

$$R(t) = \begin{cases} \dfrac{\alpha}{\alpha - 1}\left(1 - \alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}} - 1}\right), & \alpha > 0, \alpha \neq 1 \\ 1 - \alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}} - 1}, & \alpha = 1. \end{cases}$$

3.1. Hazard Rate Function

Given by is the Hazard Rate Function (HR) of random variable x for the lifetime with the NAPIW distribution.

$$h_{NAPIW}(t) = \begin{cases} \dfrac{\log(\alpha) 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}+a-(tx)^2} \left(1-e^{-(tx)^2}\right)^{-\rho b-1} \alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}}{1-\alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}}} - 1}, & \alpha > 0, \alpha \neq 1 \\ \dfrac{\log(\alpha)}{\alpha-1} 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b+a-(tx)^2}} \left(1-e^{-(tx)^2}\right)^{-\rho b-1} \alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}, & \alpha = 1. \end{cases}$$

The graphical depiction of HRF for various values of α, a, b, and ρ is displayed in Figure 2.

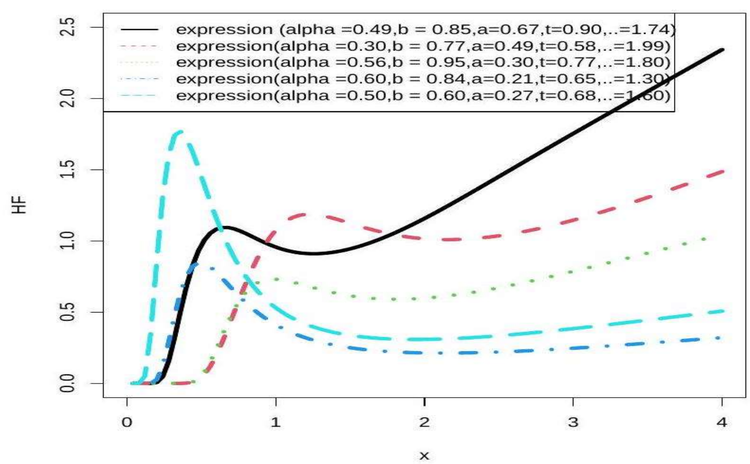

Figure 2: HF plot for new model

Table 1: Some features of NAPIW for picked values $\mathrm{b} = 6.8, \mathrm{a} = 1.5, \mathrm{t} = 5.6, \rho = 2.5, \mathrm{x} = 4$

Parameter α	RFC	HF	RHF
$\alpha = 0.2$	0.798	2.351	1.865
$\alpha = 0.49$	0.508	2.566	2.397
$\alpha = 0.99$	0.009	0.036	2.883
$\alpha = 1.46$	-1.498	-0.552	3.178

3.2. Reversed Hazard Rate Function (RHF)

The (RHF)

$$r_{NAPIW}(t) = \begin{cases} \dfrac{\psi_{s,d,j} x e^{-(d+1)(tx)^2}}{\alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}{e^{-a}}} - 1} & \alpha > 0, \alpha \neq 1, \\ \dfrac{\log(\alpha)}{\alpha - 1} 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b + a - (tx)^2}} \left(1 - e^{-(tx)^2}\right)^{-\rho b - 1} \alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}, & \alpha = 1. \end{cases} \quad (10)$$

The RHF is shown in Figure 3 for various values of α. For a given value of $\mathrm{b} = 6.8, \mathrm{a} = 1.5, \mathrm{t} = 5.6, \rho = 2.5, \mathrm{x} = 4$ and $\mathrm{t} = 0.6$, Table 1 presents the results of RFC, HF, and RHF for various values of the parameter α. It is evident that as α is rising, the values of HF and RFC are falling.

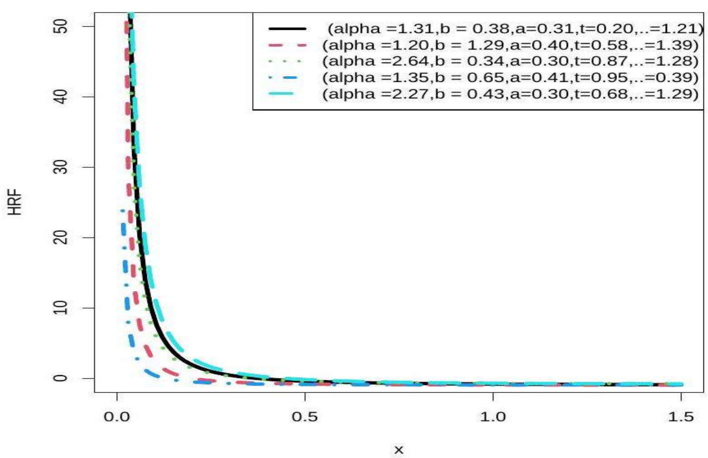

Figure 3: HRF plot for new model

3.3. Mean Residual life

In Reliability and Survival Analysis, the Mean Residual Life (MRL) function, which characterizes the aging process, is crucial. The MRL function for a lifetime random value x is provided by

$$\mu(t) = \frac{1}{R(t)} \int_t^\infty x g(x) dx - t,\ t > 0$$

$$\mu(t) = \frac{1}{R(t)} \int_t^\infty x \psi_{s,d,j} x e^{-(d+1)(tx)^2} dx - t, \quad t > 0.$$

3.4. Mean Inactivity time (MIT)

The (MIT) function is employed as a reliable metric in a number of disciplines, including survival analysis, reliability theory, and forensic science. The MIT function for the lifetime random value x is shown below.

$$\mu(t) = \frac{1}{G(t)} \int_0^\infty x g(x) dx - t,\ t > 0.$$

Table 2: Moments of NAPIW for choose values of $\mathrm{b} = 0.8, \mathrm{a} = 1.5, \mathrm{t} = 5.6, \rho = 0.5$

n	Parameter α	Mean	Variance	Skewness	Kurtosis
$\mathrm{n} = 60$	$\alpha = 0.1$	6.401	26.911	1.479	6.303
	$\alpha = 0.7$	0.953	0.813	4.233	25.608
$\mathrm{n} = 100$	$\alpha = 0.1$	9.88	52.29	1.472	5.5471
	$\alpha = 0.7$	11.687	34.28	3.289	15.934

3.5. Moment

$$\mu_r(t) = E(x^r) = \frac{1}{G(t)} \int_0^\infty x^r g(x) dx = \psi_{s,d,j} \int_0^\infty x x^r e^{-(d+1)(tx)^2} dx.$$

let

$$j = (d+1)(tx)^2 \ \Rightarrow\ x^2 = \frac{j}{(d+1)t^2},$$

$$x = \frac{\sqrt{j}}{t\sqrt{(d+1)}} \ \Rightarrow\ d(x) = \frac{dj}{2t\sqrt{(d+1)}\sqrt{j}},$$

then

$$\mu_r(t) = \psi_{r,n} \int_0^\infty x^{r+1} e^{-(d+1)(tx)^2} dx,$$

$$\mu_r(t) = \psi_{r,n} \int_0^\infty \left(\frac{\sqrt{j}}{t\sqrt{(d+1)}}\right)^{r+1} e^{-j} \frac{dj}{2t\sqrt{(d+1)}\sqrt{j}},$$

then

$$\mu_r(t) = \frac{\psi_{r,n}}{2t^{r+2}(d_+1)^{\frac{r+2}{2}}}\Gamma\left(\frac{r+2}{2}\right),$$

where $r = 1$

$$\mu_1(t) = \frac{\psi_{1,n}}{2t^3(d_+1)^{\frac{3}{2}}}\Gamma\left(\frac{3}{2}\right) = \frac{\psi_{1,n}}{2t^3(d_+1)^{\frac{3}{2}}}\sqrt{\pi}.$$

And Variance

$$\text{Var}\,(t) = E(X^2) - (E(X))^2 = \frac{\psi_n}{4t^3(d_+1)^{\frac{3}{2}}}\sqrt{\pi} - \frac{\psi_n}{2t^2(d_+1)^2}.$$

Table 2 points the moments of NAPIW for chose values of $\mathrm{b} = 0.8, \mathrm{a} = 1.5, \mathrm{t} = 5.6$, and $\rho = 0.5$ and for different values of α.

3.6. Stress-Strength Reliability

The reliability's stress-strength (supply-demand) method shows how long a component lasts when it is subjected to random stress Z and has a random strength X. When the stress applied to the component exceeds its strength, the component fails, and it will work favorably whenever $X > Z. R = \Pr\,(X > Z)$ is Consequently, a component reliability measure. It is used in a wide range of scientific and engineering fields.

The reliability R is now calculated when X and Z have independent NAPIW $(\alpha_1, \rho_1, a_1, b)$ and NAPIW $(\alpha_2, \rho_2, a_2, b)$ distributions with the same shape parameter b. The PDF of X and the CDF of Z can both be represented as

$$g_1(t) = \frac{\log\,(\alpha_1)}{\alpha_1 - 1} 2a_1 b\rho_1 x t^2 e^{-a_1\left[1-e^{-(tx)^2}\right]^{-\rho_1 b + a_1 - (tx)^2}} \left(1 - e^{-(tx)^2}\right)^{-\rho_1 b - 1} \alpha_1^{e^{-a_1\left[1-e^{-(tx)^2}\right]^{-\rho_1 b}}},$$

and

$$G_2(t) = \frac{\alpha_2^{\frac{e^{-a_2\left[1-e^{-(tx)^2}\right.2-\rho_2 b}}{e^{-a_2}}} - 1}{\alpha_2 - 1},$$

we have

$$R(t)=\int_0^\infty g_1(t)G_2(t)dt=\frac{\log(\alpha_1)}{\alpha_1-1}2a_1b\rho_1xt^2$$
$$\int_0^\infty e^{-a_1\left[1-e^{-(tx)^2}\right]^{-\rho_1 b+a_1-(tx)^2}}\left(1-e^{-(tx)^2}\right)^{-\rho_1 b-1}\alpha_1^{e^{-a_1\left[1-e^{-(tx)^2}\right]^{-\rho_1 b}}}$$
$$\frac{\alpha_2^{\frac{e^{-a_2\left[1-e^{-(tx)^2}\right]^{-\rho_2 b}}}{e^{-a_2}}}-1}{\alpha_2-1}dt.$$

By utilizing Taylor's series expansion of the function $e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}$, same as before in section NAPIW Specification.

$$R(t)=\frac{\log(\alpha_1)}{(\alpha_1-1)(\alpha_2-1)}2a_1b\rho_1xt^2$$
$$\sum_{j=d=s=0}^{\infty}\frac{\log(\alpha_1)^{j+1}\log(\alpha_2)^d(-1)^s a_1^s\Gamma(a_1bs\rho+b\rho_1+d+1)}{j_!s!\,d_!\Gamma(bs\rho_1+b\rho_1+1)}.$$

3.7. Maximum Likelihood Estimation (MLE) of R

The total likelihood function is followed by

$$\ell(a,b,\rho,\alpha)=\frac{\log(\alpha)^n}{(\alpha-1)^n}2^na^nb^n\rho^nxt^{2n}\prod_{i=1}^{n}x_ie^{-a\left[1-e^{-t^2\sum_{i=1}^n x_i^2}\right]^{-\rho b}}$$
$$e^{-t^2\sum_{i=1}^n x_i^2}\left(1-e^{-(t)^2\sum_{i=1}^n x_i^2}\right)^{-\rho b-1}$$
$$\alpha^{\sum_{i=1}^n e^{-a\left[1-e^{-(tx_i)^2}\right]^{-\rho b}}}.$$

The total log-likelihood is written by

$$L(a,b,\rho,\alpha)=n\log(\log(\alpha))+n\log 2+n\log(a)+n\log(b)+n\log\rho$$
$$+2n\log(t)+\sum_{i=1}^{n}x_i^2-a\left[1-e^{-t^2\sum_{i=1}^n x_i^2}\right]^{-\rho b}$$
$$-t^2\sum_{i=1}^{n}x_i^2-(\rho b+1)\log\left(1-e^{-t^2\sum_{i=1}^n x_i^2}\right)$$
$$-a\log(\alpha)\left[1-e^{-t^2\sum_{i=1}^n x_i^2}\right]^{-\rho b},$$

with respect to the four parameters, by computing the initial partial derivatives of the log-likelihood.

$$\frac{\partial(L)}{\partial(\alpha)}=\frac{n}{\log(\alpha)}-\frac{n}{\alpha-1}+\frac{\sum_{i=1}^{n}\ a\left[1-e^{-t^2\sum_{i=1}^{n}x_i^2}\right]^{-\rho b}}{\alpha},$$
$$\frac{\partial(L)}{\partial(\rho)}=\frac{n}{\rho}-b\left(a\left[1-e^{-t^2\sum_{i=1}^{n}x_i^2}\right]^{-\rho b-1}\right)-b\log(\alpha)\left(a\left[1-e^{-t^2\sum_{i=1}^{n}x_i^2}\right]^{-\rho b}\right),$$
$$\frac{\partial(L)}{\partial(b)}=\frac{n}{b}-\rho\log\left(1-e^{-t^2\sum_{i=1}^{n}x_i^2}\right]^{-\rho}\ \Big)-\rho\log(\alpha)\left(a\left[1-e^{-t^2\sum_{i=1}^{n}x_i^2}\right]^{-\rho b}\right),$$
$$\frac{\partial(L)}{\partial(a)}=\frac{n}{a}+\left[1-e^{-t^2\sum_{i=1}^{n}x_i^2}\right]^{-\rho b}.$$

The nonlinear equations $\frac{\partial(L)}{\partial(\alpha)}=0,\frac{\partial(L)}{\partial(\rho)}=0,\frac{\partial(L)}{\partial(b)}=0$ and $\frac{\partial(L)}{\partial(a)}=0$ are solved to get the MLE of $(\alpha,\rho,$ b, and a).

3.8. Read Data Analysis

For the sake of illustration, we offer the analysis of actual data in this section that was partially taken into account by [16. The information shows how long customers have to wait (in minutes) before receiving assistance from two different banks. Tables (3 and 4) represent the data sets.

We investigated the MLEs' behavior under uncertain conditions. For parameter, $(\alpha,\rho,\mathrm{a},\mathrm{b})$ are (0.9,0.8,1.9,2.5) respectively, and for two real sample size (60 and 100) in tables 3 and 4 from NAPIW method by using R. Table 5 displays the Mean Square Error (MSE) and bias of parameters values. From this table, it can be shown that as sample size n ascends, the MSE and the Bias for $(\alpha,\rho,\mathrm{a},\mathrm{b})$ estimations decrease.

3.9. Simulation Study

In this part, we looked at how the MLEs behaved when given unknown parameters. R is used to simulate 10,000 different random samples from NAPIW methods of different sizes (100,250,500) for the parameter values $\alpha=(0.5,0.9)$, $\rho=(1.9,0.9)$, $\mathrm{a}=(0.5,0.7)$ and $\mathrm{b}=(0.9,1.9)$. The MSE and the bias of the parameters are displayed in Table 7. The MSE and the Bias for the estimations of $(\alpha,\rho,\mathrm{a},\mathrm{b})$ are declining as the sample size n increases, as can be seen from this table.

Table 3: The time spent waiting for assistance at bank A is measured in minutes.

0.8	0.8	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7
2.9	3.1	3.2	3.3	3.5	3.6	4	4.1	4.2	4.2
4.3	4.3	4.4	4.4	4.6	4.7	4.7	4.8	4.9	4.9
5	5.3	5.5	5.7	6.1	6.2	6.2	6.2	6.3	6.7
6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6	7.7	8
8.2	8.6	8.6	8.6	8.8	8.8	8.9	8.9	9.5	9.6
9.7	9.8	10.7	10.9	11	11	11.1	11.2	11.2	11.5
11.9	12.4	12.5	12.9	13	13.1	13.3	13.6	13.7	13.9
14.1	15.4	15.4	17.3	17.3	18.1	18.2	18.4	18.9	19
19.9	20.6	21.3	21.4	21.9	23	27	31.6	33.1	38.5

Table 4: The time spent waiting for assistance at bank B is measured in minutes.

0.1	0.2	0.3	0.7	0.9	1.1	1.2	1.8	1.9	2
2.2	2.3	2.3	2.3	2.5	2.6	2.7	2.7	2.9	3.1
3.1	3.2	3.4	3.4	3.5	3.9	4	4.2	4.5	4.7
5.3	5.6	5.6	7.2	7.3	6.6	6.8	7.3	7.5	7.7
7.7	8	8	8.5	8.5	8.7	9.5	10.7	10.9	11
12.1	12.3	12.8	12.9	13.2	13.7	14.5	16	16.5	28

Table 5: Estimation parameter b by using Maximum Likelihood

NAPIW $(\alpha, \rho, \mathrm{a}, \mathrm{b})$	parameter	n	Estimate	Bias	MSE
NAPIW (0.9,0.8,1.9,2.5)	α	60	0.043	2.414	5.577
		100	0.033	2.148	2.28
	ρ	60	1.931	4.2247	2.055
		100	0.9785	3.48	1.28
	a	60	2.424	4.954	2.445

		100	1.0243	4.48	2.281
	b	60	1.983	4.954	3.432
		100	0.440	4.201	3.067

Table 6: MLE of parameters $(\alpha, \rho, \mathrm{a}, \mathrm{b})$.

NAPIW $(\alpha, \rho, \mathrm{a}, \mathrm{b})$	n	MSE (α) Bias (α)	MSE (ρ) Bias (ρ)	MSE (a) Bias (a)	MSE (b) Bias (b)
NAPIW (0.5,1.9,0.5,0.9)	100	2.185	2.283	2.257	2.623
		0.121	0.022	0.172	0.191
	250	2.113	2.236	2.346	2.566
		0.124	0.093	0.043	0.043
	500	2.125	2.158	2.313	2.496
		0.065	0.014	0.014	0.020
NAPIW (0.9,0.9,0.7,1.9)	100	0.086	0.072	0.069	0.084
		0.042	0.008	-0.054	0.009
	250	0.071	0.082	0.068	0.078
		-0.005	-0.015	-0.003	-0.032
	500	0.073	0.081	0.055	0.076
		-0.004	-0.0002	-0.001	-0.001

3.10. Fitting reliability data

In this part, we compare the NAPIW to other well-known distributions, including the APIW distribution, and inverse Weibull (IW) distributions, using real data 3 to show that the NAPIW can be a good lifetime model. The sample's parameter is numerically estimated. The MLE of the unidentified parameters and the Kolmogorov-Smirnov (K-S) statistics with their related p-values for the NAPIW and certain distributions are listed in 7 along with the results of equations (15-18), which we utilize to generate the MLEs estimate.

7 further shows that these data match the NAPIW quite closely, as evidenced by the small K-S distance and the significant p-value for the test.

Table 7: Parameter estimates and goodness-of-fit tests for the data set.

Dist.			Est.				Stat.	
	α	λ	β	ρ	a	b	$K - S$	p-Value
NAPIW	450.67	-	-	0.62	0.756	0.467	0.103	0.089
APIW	457.43	0.490	0.94	-	-	-	0.187	0.091
IW	-	13.51	0.398	-	-	-	0.421	4.822×10^{-24}

4. Conclusion

In this study, we presented our novel distribution proposal, which we dubbed New Alpha power Inverse Weibull. Many aspects of our suggested model, such as Reliability, MRL, MIT, Moments, and Stress-Strength Reliability were examined. The maximum likelihood approach is employed in the parameter estimation process. Real data as represent the time spent waiting for assistance at two banks and simulated data sets are used, and the results show that the new distribution that is presented offers more flexibility and a better fit for the data.

Availability of Data

The datasets that support the paper's results are included in the paper.

References

[1] Nelson, Wayne B. Applied life data analysis. John Wiley and Sons, 2005.

[2] Calabria, R., and G. Pulcini. "On the maximum likelihood and least-squares estimation in the inverse Weibull distribution." Statistica applicata 2, no. 1 (1990): 53-66.

[3] Calabria, R., and G. Pulcini. "Bayes 2-sample prediction for the inverse Weibull distribution." Communications in Statistics-Theory and Methods 23, no. 6 (1994): 1811-1824.

[4] Maswadah, M. "Conditional confidence interval estimation for the inverse Weibull distribution based on censored generalized order statistics." Journal of statistical Computation and Simulation 73, no. 12 (2003): 887-898.

[5] Khan, M. Shuaib, G. R. Pasha, and Ahmed Hesham Pasha. "Theoretical analysis of inverse Weibull distribution." WSEAS Transactions on Mathematics 7, no. 2 (2008): 30-38.

[6] Sultan, Khalaf S., Moshira A. Ismail, and Asmaa S. Al-Moisheer. "Mixture of two inverse Weibull distributions: properties and estimation." Computational Statistics and Data Analysis 51, no. 11 (2007): 5377-5387.

[7] De Gusmao, Felipe RS, Edwin MM Ortega, and Gauss M. Cordeiro. "The generalized inverse Weibull distribution." Statistical Papers 52 (2011): 591619.

[8] Khan, Muhammad Shuaib, and Robert King. "Modified inverse Weibull distribution." Journal of statistics applications and Probability 1, no. 2 (2012): 115.

[9] Hanook, Sharoon, Muhammad Qaiser Shahbaz, Muhammad Mohsin, and B. M. Golam Kibria. "A note on beta inverse-Weibull distribution." Communications in Statistics-Theory and Methods 42, no. 2 (2013): 320-335.

[10] Pararai, Mavis, Gayan Warahena-Liyanage, and Broderick O. Oluyede. "A new class of generalized inverse Weibull distribution with applications." Journal of Applied Mathematics and Bioinformatics 4, no. 2 (2014): 17.

[11] Aryal, Gokarna, and Ibrahim Elbatal. "Kumaraswamy modified inverse Weibull distribution: Theory and application." Applied Mathematics and Information Sciences 9, no. 2 (2015): 651.

[12] Elbatal, Ibrahim, Francesca Condino, and Filippo Domma. "Reflected generalized beta inverse Weibull distribution: definition and properties." Sankhya B 78 (2016): 316-340.

[13] Okasha, Hassan M., A. H. El-Baz, A. M. K. Tarabia, and Abdulkareem M. Basheer. "Extended inverse Weibull distribution with reliability application." Journal of the Egyptian Mathematical Society 25, no. 3 (2017): 343-349.

[14] Mahdavi, Abbas, and Debasis Kundu. "A new method for generating distributions with an application to exponential distribution." Communications in Statistics-Theory and Methods 46, no. 13 (2017): 6543-6557.

[15] Basheer, Abdulkareem M. "Alpha power inverse Weibull distribution with reliability application." Journal of Taibah University for Science 13, no. 1 (2019): 423-432.