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QUANTILE MOMENT ESTIMATION OF NEW ALPHA POWER WEIBULL FOR BANK ASSISTANCE WAITING TIME

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Abstract: The paradigm of the New Alpha power transformation, which is an original and effective strategy, was used to build the New Generalized Alpha Inverse Weibull (NAPIW) distribution, which exhibits spectacular statistical properties and has useful applications. In this paper, the classification of the NAPIW distribution is thoroughly investigated, and the basic statistical properties, such as dependability and moments, are addressed in detail. The NAPIW distribution's behavior can only be understood with the help of this information. The Linear Quantile Moment method and the Maximum Likelihood Estimation approach are both utilized to streamline practical implementations for precise parameter estimation of the NAPIW distribution. The efficacy of the proposed distribution is demonstrated through its successful application to real-world data, specifically the analysis of customer waiting times in a bank, as well as simulations, underscoring its suitability for modeling diverse datasets. Additionally, comprehensive assessments, including mean square errors for all parameters and other relevant measurements, are performed, validating the superior performance of the NAPIW distribution over existing models, and establishing it as a robust and versatile tool for researchers and practitioners in various fields.
keywords: Alpha Inverse Weibull, Linear Quantile Moment, Reliability,Inactivity time.

1. Introduction

In numerous applications, such as the dynamic components of diesel engines and various data sets, such as the durations to a breakdown of an insulating fluid subject to constant tension, the inverse Weibull (IW) distribution is crucial. For further information, see Nelson [1]. The Probability Density Function and Cumulative Distribution Function of a random variable x are specified by Inverse Weibull Distribution with parameters a, b and ρ.

$$F(x) = \frac{e^{-a[1-e^{-(tx)^2}]-\rho b}}{e^{-a}}, \tag{1}$$

and

$$f(x) = 2ab\rho x t^2 e^{-a[1-e^{-(tx)^2}]^{-\rho b + a \ (tx)^2}}(1 - e^{-(tx)^2})^{-\rho b - 1}. \quad (2)$$

Inverse Weibull Distribution research has been extensively investigated, with Calabria and Pulcini [2] studying the Maximum Likelihood and Least Squares Estimation of the inverse Weibull distribution, for instance. Inverse Weibull Distribution Bayes 2-sample prediction has been addressed by Calabria and Pulcini [3] have tackled inverse Weibull Distribution Bayes 2-sample prediction. The Inverse Weibull Distribution has been fitted to the flood data for Maswadah in [4]. Khan et al.[5] conducted some significant theoretical research on the Inverse Weibull Distribution.

Several Inverse Weibull Distribution generalizations have recently been examined by authors, including Sultan et al.[6] of two Inverse Weibull Distributions, the modified Inverse Weibull Distribution by Khan and King [8], the generalized Inverse Weibull Distribution by De Gusmao et al. [7], beta Inverse Weibull Distribution by Hanook et al. [9], Gamma Inverse Weibull Distribution by Pararai et al.[10], Inverse Weibull Distribution improved via Kumaraswamy by Aryal and Elbatal [11]. Elbatalet et al. [12] considered generalized Beta Inverse Weibull Distribution, and Okasha et al.[13] they Marshall-Olkin expanded Inverse Weibull distribution was discovered. To produce a family of distributions, Mahdavi and Kundu [14] suggested transforming the baseline (CDF) by including a new parameter. The authors of [15] analyzed the effect of (EPFS) and (EPHS) on Economic Growth (GDP) with the ARDL Cointegration test. The authors in [16] studied the application of linear regression modeling to real life data, as well as applied the theories of the proposed model to create a mathematical equation to anticipate future.

The applications of dependability with Alpha power inverse Weibull distribution, such as hazard function, mean residual life, and moments, were highlighted by Basheer in [21] and pointed out the applications of reliability with Alpha power inverse Weibull distribution such as hazard function, mean residual life, and moments. The authors of [22] explored nonparametric functional regression using the Kernel Model (funopare.kernel.cv) and KNN Model (funopare.knn.gcv) for scalar Y and functional x. To produce a family of distributions, the authors of [24] suggested transforming the baseline (CDF) by including a new parameter. The authors in [25] proposed an algorithmic application for transforming positive original responses in an academic setting. And in [20] the authors proposed a study exploring the application of the transmuted Weibull distribution for modeling lifetime data, highlighting its versatility in real-world scenarios the proposed method is known as New Alpha Power Transformation (NAPT). If $F(x)$ is a Cumulative Density Function of any distribution, then the GAP T(X) is the cumulative density distribution function.

$$G_{APT}(X) = \begin{cases} \frac{\alpha^{F(x)} - 1}{\alpha - 1}, & \alpha > 0, \alpha \neq 1 \\ F(x), & \alpha = 1 \end{cases} \quad (3)$$

the related probability density function (PDF), as well as

$$g_{APT}(X) = \begin{cases} \frac{\log(\alpha)}{\alpha-1} f(x)\alpha^{F(x)}, & \alpha > 0, \alpha \neq 1 \\ f(x), & \alpha = 1 \end{cases} \tag{4}$$

2. New Model

The New Alpha Power Inverse Weibull Distribution and a couple of its submodels are shown in this section.When $\Theta = (a, b, \rho, \alpha,)$, the distribution assigned by (3) is substituted for the Cumulative Function of the inverse Weibull distribution given by (1) in New Alpha Power to create a new distribution known as the NAPIW (x, Θ) distribution.

$$G_{NAPIW}(X) = \begin{cases} \frac{\alpha^{\frac{e^{-a[1-e^{-(tx)^2}]^{-\rho b}}}{e^{-a}}}-1}{\alpha-1}, & \alpha > 0, \alpha \neq 1 \\ \frac{e^{-a[1-e^{-(tx)^2}]^{-\rho b}}}{e^{-a}}, & \alpha = 1 \end{cases} \tag{5}$$

A PDF (corresponding Probability Density Function) is attached.

$$g_{NAPIW}(X) = \begin{cases} \frac{\log(\alpha)}{\alpha-1} 2ab\rho x t^2 e^{-a[1-e^{-(tx)^2}]^{-\rho b + a - (tx)^2}} (1-e^{-(tx)^2})^{-\rho b - 1} \\ \qquad \alpha^{e^{-a[1-e^{-(tx)^2}]^{-\rho b}}}, \quad \alpha > 0, \alpha \neq 1 \\ 2ab\rho x t^2 e^{-a[1-e^{-(tx)^2}]^{-\rho b + a - (tx)^2}} (1-e^{-(tx)^2})^{-\rho b - 1}, \quad \alpha = 1 \end{cases} \tag{6}$$

with the function $e^{-a[1-e^{-(tx)^2}]^{-\rho b}}$, we can rewrite the PDF as follows when $\alpha > 0, \alpha \neq 1$, by applying Taylor's series expansion:

$$\begin{aligned} e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}} &= \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} \left[a\left(1-e^{-(tx)^2}\right)^{-\rho b}\right]^s \\ &= \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} a^s \left(1-e^{-(tx)^2}\right)^{-\rho bs}, \end{aligned}$$

and

$$\alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}} = \sum_{j=0}^{\infty} \frac{\log(\alpha)^{j+1}}{j_{j!}},$$

then

$$g_{NAPIW}(X) = \frac{1}{\alpha-1}\sum_{j=0}^{\infty} \frac{\log(\alpha)^{j+1}}{j!} 2ab\rho t^2 x \sum_{s=0}^{\infty} \sum_{d=0}^{\infty} \frac{(-1)^s a^s}{s_! d_!} \frac{\Gamma(abs\rho+b\rho+d+1)}{\Gamma(bs\rho+b\rho+1)},$$
$$e^{-(tx)^2} e^{-d(tx)^2}$$
$$g_{NAPIW}(X) = \frac{2ab\rho t^2 x}{\alpha-1}\sum_{j=d=s=0}^{\infty} \frac{\log(\alpha)^{j+1}(-1)^s a^s \Gamma(abs\rho+b\rho+d+\ )}{j!s!d!\Gamma(bs\rho+b\rho+1)}, \quad (7)$$ Then
$$e^{-(d+1)(tx)^2}$$
$$g_{NAPIW}(X) = \psi_{s,d,j} x e^{-(d+1)(tx)^2}.$$

$$\psi_{s,d,j} = \frac{2ab\rho t^2}{\alpha-1} \sum_{j=d=s=0}^{\infty} \frac{\log(\alpha)^{j+1}(-1)^s a^s \Gamma(abs\rho+b\rho+d+1)}{j!\,s!\,d!\,\Gamma(bs\rho+b\rho+1)}.$$

Figure 1 Figure 1 shows the PDF visual depiction for the new model.

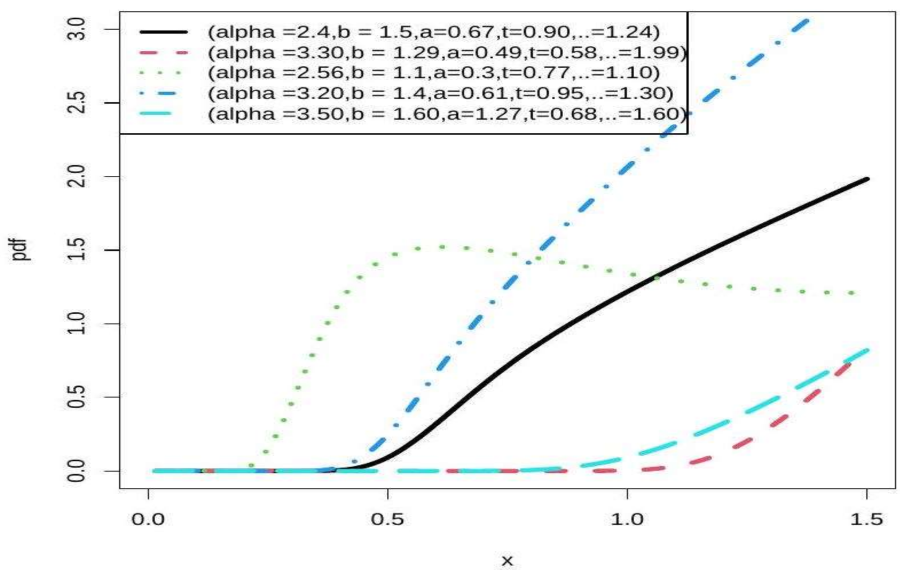

Figure 1: PDF plot for new model

3. Reliability analysis

The NAPIW Distribution's survival analysis (Reliability Function) is supplied by

$$R(t) = \begin{cases} \frac{\alpha}{\alpha-1}(1-\alpha^{\frac{e^{-a[1-e^{-(tx)^2}]-\rho b}}{e^{-a}}}-1), & \alpha>0, \alpha\neq 1 \\ 1-\alpha^{\frac{e^{-a[1-e^{-(tx)^2}]-\rho b}}{e^{-a}}}-1, & \alpha=1 \end{cases} \quad (8)$$

3.1. Hazard Rate Function

The Hazard Rate Function (HR) of random variable x with NAPIW distribution throughout the lifetime is given by.

$$h_{NAPIW}(t) = \begin{cases} \dfrac{\log(\alpha) 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right.^{1-\rho b+a-(tx)^2}} \left(1-e^{-(tx)^2}\right)^{-\rho b-1} \alpha^{e^{-a\left[1-e^{-(tx)^2}\right.^{2}-\rho b}}}{1-\alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right.-\rho b}}{e^{-a}}}-1}, \alpha > 0, \alpha \neq 1 \\ \dfrac{\log(\alpha)}{\alpha-1} 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b+a\ (tx)^2}} \left(1-e^{-(tx)^2}\right)^{-\rho b-1} \alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}, \alpha = 1. \end{cases} \quad (9)$$

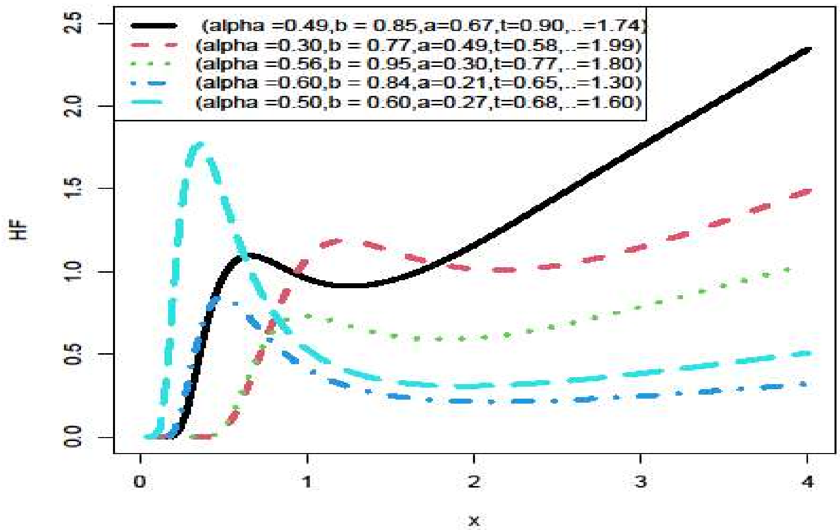

Figure 2: HF plot for new model

Table 1: Some features of NAPIW for picked values $b = 6.8, a = 1.5, t = 5.6, \rho = 2.5, x = 4$

Parameter α	RFC	HF	RHF
$\alpha = 0.2$	0.798	2.351	1.865
$\alpha = 0.49$	0.508	2.566	2.397
$\alpha = 0.99$	0.009	0.036	2.883
$\alpha = 1.46$	-1.498	-0.552	3.178

3.2. Reversed Hazard Rate Function (RHF)

The Reversed Hazard Rate Function (RHF)

$$r_{NAPIW}(t) = \begin{cases} \dfrac{\psi_{s,d,j} x e^{-(d+1)(tx)^2}}{\alpha^{\frac{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho}}}{e^{-a}}} - 1}, & \alpha > 0, \alpha \neq 1, \\ \dfrac{\log(\alpha)}{\alpha - 1} 2ab\rho x t^2 e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b + a} \, (tx)^2} \left(1 - e^{-(tx)^2}\right)^{-\rho b - 1} \alpha^{e^{-a\left[1-e^{-(tx)^2}\right]^{-\rho b}}}, & \alpha = 1. \end{cases} \tag{10}$$

Table 1 reports the outcomes of RFC, HF, and RHF for selected value of b =6.8, α=1.5,t=5.6, $\rho = 2.5$,x=4 and t=0.6, for different value of the parameter α. It is clear that the values of RFC, and HF are decreasing when α is increasing. Figure 3 HRF plot for new model

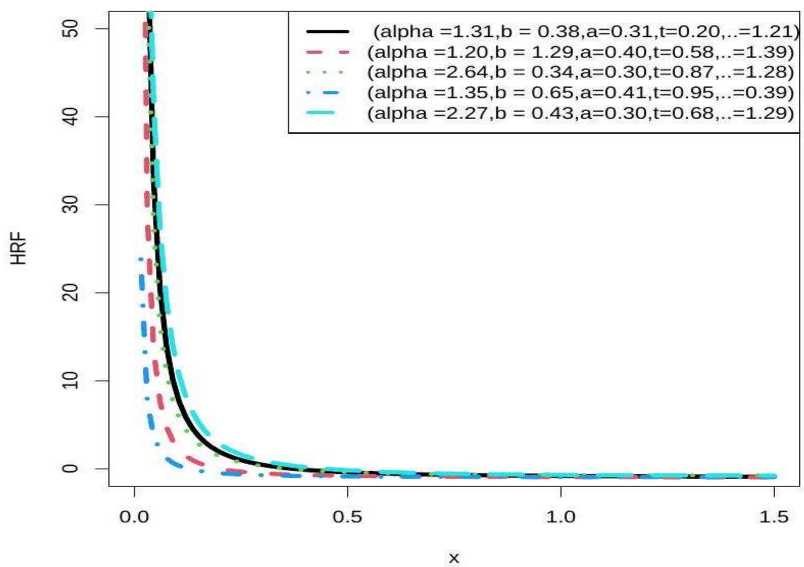

Figure 3: represents the RHF for different values of α.

3.3. Mean Residual life

In Reliability and Survival Analysis, the Mean Residual Life (MRL) function, which characterizes the aging process, is crucial. For a lifetime random variable x, the MRL[18], [23] function is given by:

$$\begin{aligned} \mu(t) &= \frac{1}{R(t)} \int_t^{\infty} x g(x) dx - t, \; t > 0 \\ \mu(t) &= \frac{1}{R(t)} \int_t^{\infty} x \psi_{s,d,j} x e^{-(d+1)(tx)^2} dx - t, \quad t > 0. \end{aligned} \tag{11}$$

3.4. Mean Inactivity time (MIT)

The (MIT) function is employed as a reliable metric in a number of disciplines, including survival analysis, reliability theory, and forensic science. The MIT function for the lifetime random value x is shown below.

$$\mu(t) = \frac{1}{G(t)} \int_0^\infty xg(x)dx - t,\ t > 0. \tag{12}$$

Table 2: Moments of NAPIW for choose values of $\mathrm{b} = 0.8, \mathrm{a} = 1.5, \mathrm{t} = 5.6, \rho = 0.5$

n	Parameter α	Mean	Variance	Skewness	Kurtosis
$\mathrm{n} = 60$	$\alpha = 0.1$	6.401	26.911	1.479	6.303
	$\alpha = 0.7$	0.953	0.813	4.233	25.608
$\mathrm{n} = 100$	$\alpha = 0.1$	9.88	52.29	1.472	5.5471
	$\alpha = 0.7$	11.687	34.28	3.289	15.934

3.5. Moment

$$\mu_r(t) = E(x^r) = \frac{1}{G(t)} \int_0^\infty x^r g(x)dx = \psi_{s,d,j} \int_0^\infty xx^r e^{-(d+1)(tx)^2} dx. \tag{13}$$

let

$$j = (d+1)(tx)^2 \ \Rightarrow\ x^2 = \frac{j}{(d+1)t^2},$$

$$x = \frac{\sqrt{j}}{t\sqrt{(d+1)}} \ \Rightarrow\ d(x) = \frac{dj}{2t\sqrt{(d+1)}\sqrt{j}},$$

then

$$\mu_r(t) = \psi_{r,n} \int_0^\infty x^{r+1} e^{-(d+1)(tx)^2} dx,$$

$$\mu_r(t) = \psi_{r,n} \int_0^\infty \left(\frac{\sqrt{j}}{t\sqrt{(d+1)}}\right)^{r+1} e^{-j} \frac{dj}{2t\sqrt{(d+1)}\sqrt{j}},$$

then

$$\mu_r(t) = \frac{\psi_{r,n}}{2t^{r+2}(d_+1)^{\frac{r+2}{2}}} \Gamma\left(\frac{r+2}{2}\right),$$

where $r = 1$

$$\mu_1(t) = \frac{\psi_{1,n}}{2t^3(d_+1)^{\frac{3}{2}}}\Gamma\left(\frac{3}{2}\right) = \frac{\psi_{1,n}}{2t^3(d_+1)^{\frac{3}{2}}}\sqrt{\pi}.$$

And Variance

$$\text{Var }(t) = E(X^2) - (E(X))^2 = \frac{\psi_n}{4t^3(d_+1)^{\frac{3}{2}}}\sqrt{\pi} - \frac{\psi_n}{2t^2(d_+1)^2}.$$

Table (2) points the moments of NAPIW for chose values of b =0.8, α=1.5,t=5.6, and $\rho = 0.5$ and for different values of α.

3.6. Stress-Strength Reliability

The reliability's stress-strength (supply-demand) model shows how long a component lasts when it is subjected to random stress Z and has a random strength X. When the stress applied to the component exceeds its strength, the component fails, and it will work favorably whenever $X > Z. R = Pr(X > Z)$ is therefore a measure of component reliability. It is used in a wide range of scientific and engineering fields.

The reliability R is now calculated when X and Z have independent NAPIW $(\alpha_1, \rho_1, a_1, b)$ and NAPIW $(\alpha_2, \rho_2, a_2, b)$ distributions with the same shape parameter b. The PDF of X and the CDF of Z can both be represented as

$$g_1(t) = \frac{\log(\alpha_1)}{\alpha_1 - 1} 2a_1 b\rho_1 x t^2 \quad e^{-a_1[1-e^{-(tx)^2}]^{-\rho_1 b + a_1 - (tx)^2}} (1 - e^{-(tx)^2})^{-\rho_1 b - 1} \alpha_1^{e^{-a_1[1-e^{-(tx)^2}]^{-\rho_1 b}}},$$

and

$$G_2(t) = \frac{\alpha_2^{\frac{e^{-a_2[1-e^{-(tx)^2}]^{-\rho_2 b}}}{e^{-a_2}}} - 1}{\alpha_2 - 1},$$

we have

$$R(t) = \int_0^\infty g_1(t)G_2(t)dt = \frac{\log(\alpha_1)}{\alpha_1 - 1} 2a_1 b\rho_1 x t^2 \int_0^\infty e^{-a_1[1-e^{-(tx)^2}]^{-\rho_1 b} + a_1 - (tx)^2} (1 - e^{-(tx)^2})^{-\rho_1 b - 1} \alpha_1^{e^{-a_1[1-e^{-(tx)^2}]^{-\rho_1 b}}} \frac{\alpha_2^{\frac{e^{-a_2[1-e^{-(tx)^2}]^{-\rho_2 b}}}{e^{-a_2}}} - 1}{\alpha_2 - 1} dt.$$

By employing the Taylor's series expansion of the function $e^{-a[1-e^{-(tx)^2}]^{-\rho b}}$, as previously described in the NAPIW Specification section.

$$R(t) = \frac{\log(\alpha_1)}{(\alpha_1-1)(\alpha_2-1)} 2a_1 b\rho_1 x t^2 \sum_{j=d=s=0}^{\infty} \frac{\log(\alpha_1)^{j+1} \log(\alpha_2)^d (-1)^s a_1^s \Gamma(a_1 bs\rho + b\rho_1 + d + 1)}{j_! s_! d_! \Gamma(bs\rho_1 + b\rho_1 + 1)}.$$

3.7. Maximum Likelihood Estimation of R

The total likelihood function is followed by

$$\ell(a, b, \rho, \alpha) = \frac{\log(\alpha)^n}{(\alpha-1)^n} 2^n a^n b^n \rho^n x t^{2n} \prod_{i=1}^n x_i e^{-a[1-e^{-t^2 \sum_{i=1}^n x_i^2}]^{-\rho}} e^{-t^2 \sum_{i=1}^n x_i^2} (1 - e^{-(t)^2 \sum_{i=1}^n x_i^2})^{-\rho b - 1} \alpha^{\sum_{i=1}^n e^{-a[1-e^{-(tx_i)^2}]^{-\rho b}}}.$$

The total log-likelihood is written by

$$\begin{aligned} L(a, b, \rho, \alpha) = \; & n\log(log(\alpha)) + n\log 2 + n\log(a) + n\log(b) + n\log\rho \\ & +2n\log(t) + \sum_{i=1}^n x_i^2 - a[1 - e^{-t^2 \sum_{i=1}^n x_i^2}]^{-\rho b} \\ & -t^2 \sum_{i=1}^n x_i^2 - (\rho b + 1)\log(1 - e^{-t^2 \sum_{i=1}^n x_i^2}) \\ & -a log(\alpha)[1 - e^{-t^2 \sum_{i=1}^n x_i^2}]^{-\rho b}, \end{aligned} \tag{14}$$

by calculating the initial partial derivatives of the log-likelihood with respect to the four parameters

$$\frac{\partial(L)}{\partial(\alpha)} = \frac{n}{log(\alpha)} - \frac{n}{\alpha-1} + \frac{\sum_{i=1}^n a[1-e^{-t^2 \sum_{i=1}^n x_i^2}]^{-\rho b}}{\alpha}, \tag{15}$$

$$\frac{\partial(L)}{\partial(\rho)} = \frac{n}{\rho} - b(a[1 - e^{-t^2 \sum_{i=1}^n x_i^2}]^{-\rho b - 1}) - b log(\alpha)(a\left[1 - e^{-t^2 \sum_{i=1}^n x_i^2}\right]^{-\rho b}\Big), \tag{16}$$

$$\frac{\partial(L)}{\partial(b)} = \frac{n}{b} - \rho\log(1 - e^{-t^2 \sum_{i=1}^n x_i^2}]^{-\rho b}) - \rho log(\alpha)(a\left[1 - e^{-t^2 \sum_{i=1}^n x_i^2}\right]^{-\rho b}\Big), \tag{17}$$

$$\frac{\partial(L)}{\partial(a)}=\frac{n}{a}+[1-e^{-t^2\sum_{i=1}^{n}x_i^2}]^{-\rho b}. \tag{18}$$

The nonlinear equations $\frac{\partial(L)}{\partial(\alpha)}=0$, $\frac{\partial(L)}{\partial(\rho)}=0$, $\frac{\partial(L)}{\partial(b)}=0$ and $\frac{\partial(L)}{\partial(a)}=0$ are solved to get the Maximum Likelihood Estimation of (α, ρ, b, and a).

3.8. Linear Quantile Moment method (L-Q)

The inverse cumulative distribution function provides a means of calculating the probability function. It is used to find the median, skewness, and kurtosis for distributions that do not have moments or have high variance values. This function is crucial for generating data for simulation studies. Assume that $X_1, X_2, \dots, X_n$ is a random sample drawn from a continuous distribution function $G_{NAPIW}(X)$ with a quantile function [19]:

$$Q(G_{NAPIW})=\sqrt{[-\ln[1-[-\ln[(\alpha-1)G_{NAPIW}(x)+1]+a]/[a\ln(\alpha)]]/(t^2\rho b)]}, \tag{19}$$

and let $X_{(1:n)} \leq . X_{(2:n)} \leq, \dots, X_{(n:n)} \leq$ denote the order statistics. where (ϵ_r) represents the Linear Quantile Moment of the random variable τ with two parameters (p,m) specified by Muolkar and Hutson [20] . Suppose that the Linear Quantile Moment for a sample of size n is as follows:

$$\hat{\epsilon}_r=\frac{1}{r}\sum_{k=0}^{r-1}(-1)^k\binom{x}{y}\hat{\tau}_{p,m}(X_{r-k:r}), r=1,2,.. \tag{20}$$

where ϵ_r refer to Quantile Moment and $0 \leq p \leq 0.5$, $0 \leq m \leq 0.5$ if

$$\hat{\tau}_{p,m}(X_{r-k:r})=p\hat{Q}_{r-k:r}(m)+(1-2p)\hat{Q}_{r-k:r}\left(\frac{1}{2}\right)+p\hat{Q}_{r-k:r}(1-m) \tag{21}$$

$$\hat{\tau}_{p,m}(X_{r-k:r})=p\hat{Q}[B_{r-k:r}^{-1}(m)]+(1-2p)\hat{Q}\left[B_{r-k:r}^{-1}\left(\frac{1}{2}\right)\right]+p\hat{Q}[B_{r-k:r}^{-1}(1-m)]. \tag{22}$$

Assume that the sample is aware of $\hat{Q}(u)$ as follows.

$$\hat{Q}(u)=\sum_{i=1}^{n}\left[(n)^{-1}k_h\left[\sum_{j=1}^{i}w_{j,n}-u\right]\right](X_{i,n}), 0<u<\infty \tag{23}$$

when k is a random variable for parameter h at the estimator (.)

$$w_{i,n}=\begin{Bmatrix}\frac{1}{2}\left(1-\left[\frac{n-2}{\sqrt{n(n-1)}}\right]\right) & i=1,n \\ \frac{1}{\sqrt{n(n-1)}} & i=1,2,\dots,n-1\end{Bmatrix} \tag{24}$$

$$k(t) = (2\pi)^{-\frac{1}{2}} exp\left(-\frac{t^2}{2}\right), h = (\frac{uv}{n})^{\frac{1}{2}}, v = 1 - u.$$

For the sample, the first four quantitative moments of the Moment-LQ method are specified as follows:

$$\hat{\epsilon}_1 = \hat{\tau}_{p,m}(X), \tag{25}$$

$$\hat{\epsilon}_2 = \frac{1}{2}\left[\hat{\tau}_{p,m}(X_{2:2}) - \hat{\tau}_{p,m}(X_{1:2}\right], \tag{26}$$

$$\hat{\epsilon}_3 = \frac{1}{3}\left[\hat{\tau}_{p,m}(X_{3:3}) - 2\hat{\tau}_{p,m}(X_{2:3} + \hat{\tau}_{p,m}(X_{1:3})\right], \tag{27}$$

$$\hat{\epsilon}_4 = \frac{1}{4}\left[\hat{\tau}_{p,m}(X_{4:4}) - 3\hat{\tau}_{p,m}(X_{3:4} + 3\hat{\tau}_{p,m}(X_{2:4}) + \hat{\tau}_{p,m}(X_{1:4})\right], \tag{28}$$

LQ- Skewnes and LQ- Kurtosis of a sample are defined:

$$LQ - Skewnes = \hat{\eta}_3 = \frac{\hat{\epsilon}_3}{\hat{\epsilon}_2},$$
$$LQ - Kurtosis = \hat{\eta}_4 = \frac{\hat{\epsilon}_4}{\hat{\epsilon}_2}.$$

LQ technique is used to produce estimates with the desired accuracy. To determine the values of the foure parameters, implicit equations are solved using a simple iteration in the R software program.

4. Application

4.1 Read Data Analysis

For illustrative purposes, we offer the analysis of actual data in this section that was partially taken into account by Ghitany et al. [17]. The information shows how long customers have to wait (in minutes) before receiving assistance from two different banks. Tables (3)and (4) represent the data sets.

Table 3: The time spent waiting for assistance at bank A is measured in minutes.

0.8	0.8	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7
2.9	3.1	3.2	3.3	3.5	3.6	4	4.1	4.2	4.2
4.3	4.3	4.4	4.4	4.6	4.7	4.7	4.8	4.9	4.9
5	5.3	5.5	5.7	6.1	6.2	6.2	6.2	6.3	6.7
6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6	7.7	8

8.2	8.6	8.6	8.6	8.8	8.8	8.9	8.9	9.5	9.6
9.7	9.8	10.7	10.9	11	11	11.1	11.2	11.2	11.5
11.9	12.4	12.5	12.9	13	13.1	13.3	13.6	13.7	13.9
14.1	15.4	15.4	17.3	17.3	18.1	18.2	18.4	18.9	19
19.9	20.6	21.3	21.4	21.9	23	27	31.6	33.1	38.5

Table 4: The time spent waiting for assistance at bank B is measured in minutes.

0.1	0.2	0.3	0.7	0.9	1.1	1.2	1.8	1.9	2
2.2	2.3	2.3	2.3	2.5	2.6	2.7	2.7	2.9	3.1
3.1	3.2	3.4	3.4	3.5	3.9	4	4.2	4.5	4.7
5.3	5.6	5.6	7.2	7.3	6.6	6.8	7.3	7.5	7.7
7.7	8	8	8.5	8.5	8.7	9.5	10.7	10.9	11
12.1	12.3	12.8	12.9	13.2	13.7	14.5	16	16.5	28

Table 5: MSE of the parameter estimations and a comparison of the two methods of estimation at the sample sizes (60,100) For the value set ($\alpha = 0.9, \rho = 0.9, a = 0.7, b = 1.9$) for real data.

MSE							
Sample size	Parametes	methods		Bias		Performance	
		MLE	LQ-moment	MLE	LQ-moment	MLE	LQ-moment
60	α	5.577	1.880316	2.3616	1.4334	3.3773	1.640898
	ρ	2.055	0.6010661	1.4335	0.7753		
	a	2.445	0.49	1.5636	0.7		
	b	3.432	3.592211	1.8526	1.8953		
100	α	2.28	1.6602	1.4926	1.2885	2.227	1.330837
	ρ	1.28	0.51105	1.1314	0.71449		
	a	2.281	0.4202	1.5103	0.6482		
	b	3.067	3.1521	1.7513	1.7752		

Table 6: Estimated parameters, AIC, BIC and HQIC for the real data set of the two methods of estimation at the sample sizes (60,100) For the value set ($\alpha = 0.9, \rho = 0.9, a = 0.7, b = 1.9$) for real data.

real data		
	methods	Statistics

Sample size		AIC	BIC	HQIC	Skewness	Kurtosis	Reliability
60	MLE	1062.754	1088.234	1071.111	0.8082323	2.050006	0.8914522
	LQ-moment	828.4359	844.7878	413.0153	0.5850625	1.509119	0.8947514
100	MLE	820.5055	831.1426	824.630	0.510881	2.342619	0.8888389
	LQ-moment	561.2449	585.8441	570.2965	0.428	1.936	0.8929066

We investigated the MLEs' and LQM behavior under uncertain conditions. For parameter, (α,ρ,a,b) are (0.9,0.8,1.9,2.5) respectively,and for two real sample size (60 and 100) in table (3)and table (4) from NAPIW method,The mean values (6.401667, 9.887) and median values (5, 8.1) were calculated for two sets of real data with sample sizes of 60 and 100, respectively. The aim was to determine whether the data follows the NAPIW method with four parameters ($\alpha = 0.9, \rho = 0.9, a = 0.7, b = 1.9$). To test this, a goodness-of-fit test was performed, which included the Kolmogorov-Smirnov test for the two sample sizes. The resulting p-values were 0.2756 and 0.896, respectively. The null hypothesis (H0:data distributed NAPIW method), whereas the alternative hypothesis (H1:data does not distribute NAPIW method). We fail to reject the null hypothesis because the p-values are both greater than 0.05, and we conclude that the data is consistent with the NAPIW method. The function of cumulative distribution function for the bank A data is shown in Figure (4)by using R. Table 5 displays the Mean Square Error (MSE) and bias of parameters values. From this table, it can be shown that as sample size n increases, the MSE and the Bias for (α,ρ,a,b) estimations decrease.Table (6) displays the corresponding AIC (Akaike information criterion), BIC (Bayesian information criterion), and HQIC (Hannan Quinn information criterion) values. We can see that the NAPIW distribution has the smallest AIC, BIC, and HQIC. So, when using the LQM method to estimate parameters, one can conclude that the NAPIW distribution is the best fit for this data set.

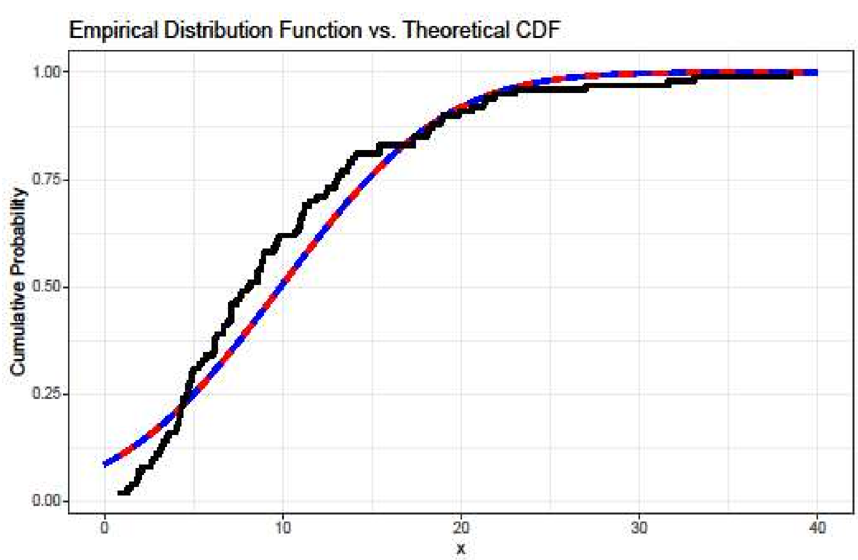

Figure 4: Cumulative distribution function for the bank Al data.

4.2. Simulation Study

In this section, we examined the behavior of the MLEs with unknown parameters. For the parameters α, ρ, a and b, R is used to simulate 10,000 distinct random samples from NAPIW methods of various sizes (100, 250, 500). Table (7) presents the Mean Square Error (MSE) and the parameter bias. This table shows that as sample size n increases, the MSE and the Bias for the estimations of (α,ρ,a,b) decrease.

Table 7: MSE of the parameter estimations and a comparison of the two methods of estimation at the sample sizes (100,250,500) For the initial value set ($\alpha = 0.5, \rho = 1.9, a = 0.5, b = 0.9$).

MSE					
Sample size	Parametes	methods		Performance	
		MLE	LQ-moment	MLE	LQ-moment
	α	2.185	0.3745095		
	ρ	2.283	1.489228	2.337	0.8226078
100	a	2.257	0.25		
	b	2.623	1.176694		
	α	2.113	0.3832199		
	ρ	2.236	0.8599968	2.301	0.6986193
250	a	2.234	0.2492		
	b	2.566	1.30126		
	α	2.125	0.0699		
	ρ	2.158	1.588186	2.273	0.5378004
500	a	2.313	0.24254		
	b	2.496	0.243099		

5. Conclusion

In this research, we introduced a unique distribution concept called the ”New Alpha Power Inverse Weibull.” Our proposed model underwent comprehensive analysis, covering various crucial aspects such as Reliability, Mean Residual Life, Mean Inactivity Time, Moments, and Stress-Strength Reliability. To estimate the parameters of our model, we adopted the maximum likelihood approach, a widely used statistical technique. Both real-world data and simulated datasets were employed in our study to evaluate the performance of the new distribution. The outcomes revealed that our novel distribution exhibits enhanced flexibility and provides a superior fit to the data compared to existing models. These findings underscore the potential practical applications of the New Alpha Power Inverse Weibull distribution in diverse fields that involve modeling and analyzing data with complex characteristics.

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